UPPSALA UNIVERSITET
NATURGEOGRAFISKA
INSTITUTIONEN
AVDELNINGEN FÖR HYDROLOGI

ANDERS WILLÉN

A STOCHASTIC HYDROLOGY APPROACH TO THE OPERATION OF SURFACE WATER RESERVOIRS

Part 2: Optimization of hydro-electricity systems

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PREFACE

The topic of this thesis was suggested to me by my supervisor professor Erik Eriksson, who has given me much valuable guidance and with whom I have had many inspiring discussions. The data values from Korsvattnet, Stora Mjölkvattnet and Landösjön were received from Indalsälvens Vattenregleringsföretag (The River Indalsälven Regulation Company), where I have had valuable discussions with several engineers, special credit is given to Eng. Hilding Hedqvist. Also obtained from the regulation company are (amongst other things) unpublished material concerning the Oldå Power Plant; fig 3 (except the location map) is mainly drawn from this material.

The manuscript has been critically read by Mr Abraham Simeonidis and Mr Björn Sundblad, both at Division of Hydrology, Uppsala.

Linguistic revision was done by Jane Mortimer, Surahammar.

I express my gratitude to all these people and to Indalsälvens Vattenregleringsföretag.

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Uppsala, September 1978

Anders Willén

ABSTRACT

A STOCHASTIC HYDROLOGY APPROACH TO THE OPERATION OF SURFACE WATER RESERVOIRS (In English).

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This thesis deals with stochastic modelling in hydrology applied to the operation of multi-reservoir systems. The un-predictability of future inflows is a fundamental feature of this problem and therefore this is a problem of stochastic optimization. Stochastic models of inflow and runoff are developed and tested in various ways. Seasonality is very marked in the records used in this study, with high flows in spring owing to snow-melt, and low flows in winter owing to snow accumulation. Two kinds of models are used for the non-spring seasons. In the Rec-Ris model, the idea is to take into account the quite different mechanisms involved during the r e c ession and r i s ing parts of runoff. The other model is based on conditional probability distributions, and it is applied to the inflow at two sites in central Sweden. Here the probability distribution of next inflow is conditioned by present inflow. In the spring, one model first determines the total amount of inflow during some weeks and then this amount is distributed over these weeks. In the other model we first make a p r e d i c t i o n of total runoff during some pentades (pentade = five days) in the spring, and then generate the runoff pentade by pentade based on this prediction. Our stochastic models of inflow are applied in the optimization of a 5-reservoir system regulated for hydropower production. This is done by Monte-Carlo generations of synthetic series, from which probabalistic forecasts of future inflow are derived.

A chance-constrained optimization model is then applied to the operation of our reservoir system. In this model the operation is determined for the whole system for a number of weeks ahead, maximum 52 weeks ahead. Thereafter a simulation model is applied, where the real inflow is used as input, and some changes in the releases are made. Using the state derived from this simulation model and new forecasts of inflow, re-optimization is applied to the rest of the planning period, and the simulation model is applied, etc.,

(KEY WORDS: Inflow modelling; Runoff modelling; Monte-Carlo simulation; Stochastic optimization; Multi-reservoir systems; Stochastic modelling).

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1 INTRODUCTION

This is the second and last part of a Ph. D thesis on stochastic modelling and optimization in hydrology.

In part one (Willén 1979) we dealt with stochastic modelling with emphasis on short term inflow and runoff, and herein we deal with stochastic optimization applied to the long-term regulation of a multireservoir system operated for hydro-power production.

The stochastic models of weekly inflow derived by Willén (1979) are used in the optimization of the releases.

In this thesis inflow means local net inflow and it is defined for a regulated reservoir by the equation

$$S_t = S_{t-1} + Q_t - X_t + XU_t \qquad (1)$$

S_t Storage in the regulated reservoir at end of time step t.
Q_t Inflow during time step t.
X_t Release from the reservoir during time step t.
XU_t Total amount of upstream release that enters the reservoir during time step t.

From eq (1) it is obvious that Q is not equal to the sum of water entering the reservoir by the river(s). Instead it is the net effect of this sum, evaporation and groundwater inflow (which may be positive or negative).

By using equations like (1) in reservoir operation studies, we try to separate the behaviour of reservoirs into two distinct parts; the natural process, Q, and the human controllable, X and XU.

This decomposition between man and nature is not without objections. In reality large storage may increase evaporation and seepage to groundwater (or decrease groundwater inflow), which means that Q decreases with increasing storage. That is inflow is not independent of X and XU. In arid regions the evaporation effect may be considerable and make the decoupling between man and nature impossible.

But in the humid climates in the Swedish mountains it is reasonable to consider Q independent of X and XU. This approximation is made throughout this paper when dealing with the author's own models.

Chapter 2 contains a general discussion of optimization techniques in water resources systems with emphasis on the operation of existing reservoir systems.

In chapter 3 a few examples of very large systems and some special methods applied to enable analysis of such systems is presented.

The following chapter is a special treatment of operation of hydro-electricity systems.

In chapter 5 the author´s own models of long-term regulation is presented, and chapter 6 contains a final discussion.

2 OPTIMIZATION IN WATER RESOURCES; SOME GENERAL ASPECTS AND A REVIEW

2.1 INTRODUCTION

This is a very broad topic and the literature is indeed very scattered. Not only do examples appear under rather different key words in the scientific literature, but many good methods are unpublished and exist as internal PM:s at engineering firms, regulation companies etc.

The main theme of this paper is the operation of an existing system of surface water reservoirs, and the emphasis is on long-term regulation for hydro-power production. Other kinds of water resources optimization problems will also be touched upon in this chapter.

The field of optimization is very old, but the advent of high-speed computers has resulted in a new trend; the computational mathematical approach by digital computers. The optimization problems discussed in this paper are very seldom solvable by analytical methods, and we have to apply numerical techniques. When dealing with real optimization problems, we have to decide both the objective(s) and the method of achieving it. In most real cases we have several objectives to optimize, i.e. multi-objective optimization. But the only kind of multi-objectives dealt with in this paper are those arising from the random character of future inflows.

There exist a few standard "formats" of single-objective optimization problems which are briefly described below, for general treatments see cited references.

Linear Programming (LP):

Maximize $$Z = \sum_{i=1}^{n} c_i \cdot x_i \qquad (2)$$

subject to the constraints

$$\sum_{i=1}^{n} a_{ji} \cdot x_j \begin{matrix} \geq \\ = \\ \leq \end{matrix} b_j \qquad j = 1,2,\ldots,m \qquad (3)$$

$$x_i \geq 0 \qquad (4)$$

where x_i ($i = 1,2,\ldots,n$) are variables to determine by the optimization procedure, a_{ji}, c_i and b_j are parameters which are independent of x. The signs in eq (3) may be any of the three examples shown. By multiplying eq (2) by minus one we obtain a minimization problem.

For LP efficient computer codes exist, which easily can handle rather large problems, see for example Lasdon (1970) and Thesen (1978).

A serious limitation of LP is that all constraints and the objective function must be linear, while indeed most real problems are non-linear both in the objective and in the constraints.

Non-linear Programming (NLP) is a more general technique which does not have these drawbacks. It can be written as

$$\text{Maximize } Z = g(x_1, x_2, \ldots, x_n) \tag{5}$$

subject to

$$h_j(x_1, x_2, \ldots, x_n) \begin{matrix} > \\ = \\ < \end{matrix} 0 \qquad j = 1,2,\ldots,m \tag{6}$$

and it includes LP as a special case. Although computer codes for NLP exist, the computer time required to solve a NLP-problem is usually much larger than that required for a LP of the same size, and large NLP:s may be computationally infeasible. NLP is treated in for example Lasdon (1970), Lootsma (1972) and Murray (1972).

Quadratic Programming (QP) requires linear constraints (as in eq (3)) but quadratic terms of the decision variables are allowed in the objective function, thus QP is a special case of NLP.

Because most real problems are very complex and also contain several conflicting objectives, it is very unusual that one can just solve one LP, QP or NLP and then know "what to do". Instead we mostly have to solve several mathematical optimization problems and then combine the results obtained with insight into the real complex problem and maybe also apply simulation studies. This means that we may state that often: "In LP the serious approximations are the linear objective and linear constraints while in NLP the serious approximation is the many variables not taken into account."

Another method is Dynamic Programming (DP) which is based on Bellman´s Principle of Optimality (from Bellman 1957): "An optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision".

In a dynamic programming formulation, the optimal decisions are obtained by solving the equations

$$f_n(S) = \underset{D}{\text{Opt}} \left\{ R_n(D) + f_{n-1}(S') \right\} \tag{7}$$

where

$f_n(S)$ return obtained if the state of the system is S and it is operated optimally when there are n stages to the end of the planning period

$R_n(D)$ benefit obtained in current time step due to decision D

S' state of the system at the end of the time period, if the state at the beginning was S and the decision was D

D decision

In these relations it is understood that only feasible values of S and D are considered.

In order to use DP we must be able to define a value function for current decision, $R_n(D)$, which is dependent on decisions in other time steps through the state only. S and D may be scalars or vectors, and they must be discretized into finite numbers of values.

The form of DP shown here is the Backward Dynamic Programming formulation, and here one starts at the end of the planning period, where we only have D for the last time step to determine. That is we start by solving

$$f_1(S) = \underset{D}{\text{Opt}} \left\{ R_1(D) \right\} \tag{8}$$

for all values of S and search for the optimum amongst the values of D. Then we proceed backwards and solve eq (7) for all values of S for $n = 2, 3, \ldots$.

Having done this the optimal policy is found by "backtracking", i.e. in the table of optimal D we choose the optimal decision in the first period, where we use the initial state as S. Then we compute the new state, S', that results from this D and proceed to next period etc.

In eq (7) n corresponds to time, but it is also possible to use DP to optimize over, for example, a line, and then n may denote a line segment.

DP can handle objective functions and constraints of many kinds, and a merit that is important in some applications is that tight constraints tend to reduce computer time, while in LP or NLP they usually increase computer time.

A general review of optimization methods in water resources is contained in Buras (1972).

2.2 APPLICATIONS OF DYNAMIC PROGRAMMING

When DP is applied to the operation of a single reservoir, we usually choose storage as S and release as D.

In case of multireservoir systems both state and decision will be multidimensional. If, in addition, there are many time steps (the planning period consists of many time steps) the computer memory requirement will be enormous. This was called "the curse of dimensionality" by Bellman and it is a problem when applying DP to multireservoir systems.

Another problem is the randomness of hydrologic quantities, which means that uncertainty of future water availability must be taken into account in one way or another in water resources optimization.

One way is Explicit Stochastic Dynamic Programming (ESDP), where the expected result is optimized. This approach to water resources was first applied by Little (1955) to a one-reservoir hydro-electricity problem. The description of the method below closely follows Butcher-Fordham (1970) who applied ESDP to a multi-purpose reservoir.

s_i storage at the start of the i:th time step

q_i the flow into the reservoir during the i:th time step

d_i release during the i:th time step

$R(d_i)$ return obtained from releasing d_i units of water in the i:th time step

$P(q_i \mid q_{i+1})$ transition probabilities relating the flow in the i:th time period with flow in the (i+1):th time period

e_i evaporation during time step i

$f_i(s_i, q_{i+1})$ expected return from the optimal operation of a system which has i time periods to the end of the planning period

Here the time steps are indexed in reverse order, i.e. i=1 corresponds to last time step. Eq (7) is written

$$f_i(s_i, q_{i+1}) =$$

$$\max_{d_i} \left[R(d_i) + \sum_{q_i = 0}^{q_i = q_i \max} P(q_i \mid q_{i+1}) \cdot f_{i-1}(s_i + q_i - d_i - e_i, q_i) \right] \qquad (9)$$

In eq (9) the state is defined by the vector (s_i, q_{i+1}), i.e. by both storage and last inflow. The optimal value of d must be found for all states, which means that this approach requires more computer memory and time than the deterministic version. It should be noticed that all probabalistic information on flows that is needed in this formulation is the Markov matrix (matrices) of transition probabilities, P, compare chapter 3 in Willén (1979).

The merits of ESDP are that the uncertainty of future flows are considered as well as probabalistic forecasting through the Markov chain(s) P. In most cases the optimal d_i will be different for different q_{i+1}, i.e. last flow.

An alternative to ESDP is Implicit Dynamic Programming (ISDP) which is also called Monte-Carlo Dynamic Programming. Here the first step is to generate a set of inflow series of suitable length by Monte-Carlo methods. The deterministic DP is applied to each of these series. Finally the operation policy is determined by studying the relationships between the optimal decisions and some parameters (for example storage and last inflow) obtained in these deterministic programs. This approach was applied by Young (1966) who in the first step generated one long sequence of flows.

The merit of this approach is that the size of the algorithm used in the deterministic optimization is directly proportional to the number of reservoirs in the system, while it grows exponentially in the ESDP approach.

The difficulties are the choice of relations to use in the last step and the high number of generated flow values required in order to get reliable results.

Forecasts of future flow values are used implicitly in this approach, because in the last step one usually includes last q among the variables determining the operation policy and because of the persistence q carries information on future flow. ESDP optimizes the expected result and does not generally consider the risk of a very bad result at all. The probabilities of very bad results may be obtained by testing the "optimal" policy on flow sequences obtained by random methods. Methods of handling risks in ESDP are discussed by Askew (1974a, 1974b, 1975, 1978) and Rossman (1977), in some cases chance-constraints can be directly included in ESDP.

McKerchar (1975) applied ISDP to a multireservoir hydroelectricity system.

Turning to the "curse of dimensionality" methods have been developed to overcome it. A usual idea is to work as follows:

i) Compute a feasible sequence of decisions (D) in eq (7) and for each stage define a neigbourhood of the value of D chosen

ii) Solve a restricted DP where we in each stage restrict the search for optimal D to the neighbourhood defined above

iii) If step ii) led to any changes, define a neighbourhood to the new D:s and go to step ii). Otherwise stop, we have completed the solution.

Step ii) can be compared with the ordinary DP-technique, where we search for optimum amongst all values of D and store the optimal D and result for all states.

When using the modified technique the possibility of being trapped at local minima should be investigated however. As examples of these and related methods, see for example Hall et al (1969), Trott-Yeh (1973), Harboe (1977), Nopmonggol-Askew (1976) and Heidari et al (1971).

Another way to reduce the computational requirements was applied by Cro-

ley (1974a, b). The idea is that when optimizing the operation for a long time period, we usually are most interested in the decisions for the first time step(s). Considering the later time steps, we usually have the possibility of modifying the operation taking into account the new information which will be available at this time. Sometimes the decisions of the first time step(s) may be determined approximately by considering an optimization problem of reduced time horizon, which reduces computer time. When using this approach, special arrangements usually must be made in the computer algorithm, however, in order to prevent having an empty reservoir at the end of the reduced time span. ASO (Alternate Stochastic Optimization) was developed by Croley (1974b) as a method specially suited for real-time operation of a stochastic system. The procedure works as follows:

i) Input realizations for the entire planning period are generated by a suitable Monte-Carlo method

ii) For each random input series a deterministic optimization technique is applied, but only the first stage decision is noted

iii) From this sample of first stage decisions an empirical probability distribution is estimated and by a suitable rule a decision is chosen

iv) The second stage decision is determined by repeating the data generations and optimizations with new values for system state, past inflows, past decisions etc, over the remaining operation horizon (one stage less than before).

The process is repeated stage-by-stage until the last decision is determined at the end of the planning period. ASO used forward Dynamic Programming, and the decision determined in iii) is used in real-time operations.

Russell (1970) and Arunkumar (1975) used analytical methods in Dynamic Programming applied to problems with random inflows.

Other examples of DP are given in chapter 4 of this report.

2.3 APPLICATIONS OF NON-LINEAR PROGRAMMING

Muspratt (1973) applied NLP to the operation of a hypothetical reservoir. The random features were taken into account by the three steps.

i) Generate sets of random variates for all stochastic parameters by a suitable Monte-Carlo method.

ii) Optimize by deterministic NLP.

iii) Repeat these steps until a large enough sample is obtained.

This approach gives the probability distribution of the optimal result con-

ditioned on perfect knowledge of future values of the random variables in each case. But the method can be combined with analysis of interrelations between optimal decisions and some parameters of the process (e.g. storage and last inflow) as in ISDP.

Colorni and Fronza (1976) considered the problem of determination of monthly contract volumes to be released by a reservoir of given capacity. Their model is

$$\underset{w,s,\alpha}{\text{Max}} \quad Z = \sum_{j=1}^{12} (f_j(w_j) - L(\alpha)) \tag{10}$$

subject to

$$m_j \leq s_j \leq C - V_j \tag{11}$$

$$s_{j+1} - s_j + w_j \leq a_j^{*\alpha} \tag{12}$$

$$w_j \geq 0 \tag{13}$$

$$j = 1,2,.....,12$$

$$s(13) = s(1) \tag{14}$$

where

w_j contract volume to be released during month j

$f_j(w_j)$ net benefit from reservoir operation during month j

s_j storage at beginning of month j

a_j net inflow during month j

v_j given minimum freeboard at the beginning of the j:th month in order to meet flood control requirements

m_j given minimum admissible value of s_j

$a_j^{*\alpha}$ is defined by the relation

$$\text{Prob}(a_j \leq a_j^{*\alpha}) = \alpha$$

Derivation of equations like (12) is found in section 2.4 in this paper.

$L(\alpha)$ is the yearly risk loss associated with α.

In eq (12) it is stated, that the probability of not being able to release the amount w_j while maintaining storage within the desired limits, must not exceed α.

Shih et al (1974) applied chance-constrained Quadratic Programming to the management of urban return flows. Their approach takes into account the reliabilities of the treatment plants.

2.4 APPLICATIONS OF LINEAR PROGRAMMING

Turning to LP an immediate question is: "Is it at all meaningful to apply linear programming in our non-linear world?" A basis for using LP in reservoir operation studies is shown by a simple theoretical example.

Consider a single reservoir operated for hydro-power production, where the release enters a turbine directly. We assume perfect knowledge of future inflows, and the head of the plant is constant.

Q_t inflow during time step t

X_t release during time step t

S_t storage at end of time step t

$Smin_t, Smax_t$ lower and upper limits of S_t

$Xmin_t, Xmax_t$ lower and upper limits of X_t

c_t price of power produced during time step t

p factor converting release into power (p is a constant)

S_o initial storage

NT number of time steps in the planning period

A natural LP-formulation is

$$\text{Max } Z = \sum_{t=1}^{NT} c_t \cdot p \cdot X_t \tag{15}$$

subject to

$$Smin_t \leq S_t \leq Smax_t \tag{16}$$

$$Xmin_t \leq X_t \leq Xmax_t \tag{17}$$

$$S_t = S_{t-1} + Q_t - X_t \tag{18}$$

The last constraint is the simple continuity equation.

Certainly this simple example is unrealistic in many ways, but it can serve as a basis for more realistic LP-models, this is done in Roefs and Bodin (1970).

A serious drawback of the model is the extreme sensitivity of X on c. This means that even if c_k is just slightly higher than c_{k-1}, the algorithm will try to put $X_k >> X_{k-1}$, maybe even choose $X_k = Xmax_k$ and $X_{k-1} = Xmin_{k-1}$. Such a release (and production) sequence is probably unacceptable both for the power company and the environmentalists.

And in most real cases we do not know the future inflows for more than one or two weeks, which means that we have a stochastic optimization problem.

Some basic approaches to stochastic optimization were described in conjunction with Dynamic Programming, and they are also possible to apply when using Linear Programming.

Another approach to uncertainty is to choose a very "bad" sequence of inflows from the historical record, and optimize the operation for this sequence. If the purpose is flood control the bad sequence consists of high inflows, while we choose a critical sequence of low flows if hydro-power production is studied. The drawback of this approach is, that the operation policy which is good for the critical sequence may be rather bad for normal flows. Consider for example the case with a critical period of low flows! The optimal operation policy for this sequence may be such that it would lead to flooding problems or at least spills in case of normal flows.

Another matter of discussion is if it is enough to study the critical flows that are contained in the historical series, or if we by Monte-Carlo methods also should study synthetic series. Some Monte-Carlo methods are described in Willén (1979). Some approaches to stochastic optimization are discussed in Loucks (1969). One of them is

$$\text{Max } Z = \sum_k \sum_i \sum_d \sum_{t=1}^{n} f(k,i,d,t) \cdot P_{k,i,d,t} \tag{19}$$

subject to

$$\sum_k \sum_i \sum_d P_{k,i,d,t} = 1 \tag{20}$$

$$P_{k,i,d,t} \geq 0 \tag{21}$$

$$\sum_e P_{L,j,e,t+1} = \sum_k \sum_i \sum_{d \atop (k+i-d=L)} P_{k,i,d,t} \cdot P^{t}_{i,j} \tag{22}$$

where

$P_{k,i,d,t}$ the joint probability of releasing an amount d, while at storage k with inflow i occurring at time t

$P_{L,j,e,t+1}$ the joint probability of releasing an amount e, while at storage L with inflow j occurring at time t+1

$P^{t}_{i,j}$ the transition probability of having inflow j at time t+1 given the inflow at time t is i

$f(k,i,d,t)$ return obtained by releasing d at time t when storage is k and inflow i.

In this optimization model all variables are discretized into finite numbers of values. In eq (22) the summation on the right side is over all combinations of initial storage, k, inflow i, and release d, that would result in a final volume L at time step t.

Then the operating policy is determined by

$$P_t(d \mid k,i) = P_{k,i,d,t} / \left(\sum_d P_{k,i,d,t}\right) \tag{23}$$

Thus even the optimal policy may be random! However one often obtains (Loucks 1969) what is called a "pure policy" with

$$P_t(d \mid k,i) = \begin{cases} 1 & \text{if } d=d^* \\ 0 & \text{else} \end{cases} \tag{24}$$

$P_t(d \mid k,i)$ is the conditional probability of releasing d given k and i.

This version can handle non-linear return functions and it optimizes the expected result. Even if risks are not considered, chance-constraints are easily added to the set of constraints. If we for example do not like low storage to occur very often, we may add the restriction

$$P_{k,i,d,t} \leq \beta \tag{25}$$

for values of d such that the probability is considered too large for having k+i-d < a desired minimum value of storage, β may even be put equal to zero, which means that for this (k,i,d) eq (25) reduces to the equality constraint

$$P_{k,i,d,t} = 0 \tag{26}$$

It should be noticed that here we do not need generated flow series but just the Markov matrices

$$P^t_{i,j}$$

in order to perform optimization (compare chapter 3 in Willén (1979)). The number of constraints is proportional to the product of ranges of inflow, ranges of storage and number of time steps, and this means that multi-reservoir problems usually become very large.

A discussion of when a random policy is better than a deterministic one is found in Rossman (1977).

ReVelle et al (1969) applied a specific form of release policy, Linear Decision Rule (LDR), to reservoir management. The stochasticity of inflows is taken into account by expressing the restrictions in chance-constrained form. Monthly operation is explicitly discussed below, transferring the description to say weekly operation involves no difficulties. First some notations.

X_t release during time step t

S_t storage at end of time step t

i remainder of t/12

R_t inflow during time step t.

The release is determined by

$$X_t = S_{t-1} - b_i \tag{27}$$

where b_i (i=1,2,.....,12) are parameters to determine.

The continuity equation is written

$$S_t = S_{t-1} - X_t + R_t \tag{28}$$

Substituting eq (27) into eq (28) gives

$$S_t = b_i + R_t \tag{29}$$

and using a similar relation for S_{t-1} we can convert eq (27) to

$$X_t = R_{t-1} + b_{i-1} - b_i \tag{30}$$

The objective was

$$\text{Min } C$$

where C is the capacity of the reservoir.

We have some constraints on releases and storage, which may be due to for example legal restrictions, technical limits, water demand or flood control. The restrictions are formulated as chance-constraints.

i) The freeboard $C-S_t$ at end of period t must be greater than v_i.

$$\text{Prob}(C-S_t < v_i) \leq \alpha_1 \tag{32}$$

with α_1 a small number.

ii) The storage at end of period t must exceed the minimum storage Smin

$$\text{Prob}(\text{Smin} > S_t) \leq \alpha_2 \tag{33}$$

iii) The release in period t must exceed Xmin_i

$$\text{Prob}(\text{Xmin}_i > X_t) \leq \alpha_3 \tag{34}$$

iv) The release must not exceed Xmax_i

$$\text{Prob}(\text{Xmax}_i < X_t) \leq \alpha_4 \tag{35}$$

By using eq (29), eq (33) may be written

$$\text{Prob}(b_i + R_t < \text{Smin}) \leq \alpha_2$$

or

$$\text{Prob}(\text{Smin}-b_i > R_t) \leq \alpha_2$$

This expression is equivalent to

$$\text{Smin}-b_i \leq R_t^{*\alpha_2} \tag{36}$$

where we in general define $R_t^{*\alpha}$ by the relation

$$\text{Prob}(R_t \leq R_t^{*\alpha}) = \alpha \tag{37}$$

In the same way the other constraints are replaced by

$$C-b_i \geq R_t^{*(1-\alpha_1)} + v_i \tag{38}$$

$$b_{i-1}-b_i \geq \text{Xmin}_i - R_{t-1}^{*\alpha_3} \tag{39}$$

$$b_{i-1}-b_i \leq \text{Xmax}_i - R_{i-1}^{*(1-\alpha_4)} \tag{40}$$

Here b_i is unrestricted in sign, but transformation to the ordinary LP-format (eqs (2)-(4)) is easily done by replacing b_i by

$$b_i = b_i' - b_i'' \tag{41}$$

where b_i' and $b_i'' \geq 0$.

LDR was initially developed for design problems, but in later developments other objective functions are also applied, for example in cases where the objective is a function of the releases. See for example ReVelle-Kirby (1970), ReVelle-Gundelach (1975). Persson (1977) applied LDR to a system of 22 reservoirs with the objective of maximizing the minimum releases. For other applications to multireservoir systems see for example Nayak-Arora (1971) and LeClerc-Marks (1973). In many cases we do not have to resort to ordinary LP-solution techniques, but more simple methods can be applied (Eastman-ReVelle 1973, Alemany et al 1973, Gundelach-ReVelle 1973, Sobel 1975 and Persson 1977).

Another approach to chance-constrained programming was applied by Curry et al (1973). Basically their model is as follows.

$$\text{Min } Z = H(X) \tag{42}$$

where

$$X = (X_1, X_2, \ldots\ldots, X_N)$$

N number of time steps in the planning period
H cost function which is linear in X

Otherwise we use the same notations as done when describing LDR.

As before the continuity equation is written

$$S_t = S_{t-1} + R_t - X_t$$

If S_o is the initial storage, we can replace eq (28) with

$$S_t = S_o + \sum_{k=1}^{t} R_k - \sum_{k=1}^{t} X_k \tag{43}$$

Here X is pre-determined at the start of the planning period, so we have

$$Xmin_t \leq X_t \leq Xmax_t \tag{44}$$

as a deterministic restriction in this optimization model. But the constraints on storage are still chance-constraints.

Smax maximum allowed value of S

$$Prob(S_t < Smin_t) \leq \alpha_1 \tag{45}$$

$$Prob(S_t > Smax_t) \leq \alpha_2 \tag{46}$$

Using eq (43) we rewrite eq (45)

$$Prob(S_o + \sum_{k=1}^{t} R_k - \sum_{k=1}^{t} X_k < Smin_t) \leq \alpha_1 \tag{47}$$

These constraints differ from the corresponding ones in LDR in that they involve a sum of random variables

$$(\sum_{k=1}^{t} R_k)$$

But assuming knowledge of the stochastic process (including the parameters) governing R, we can compute the probability distributions of these sums, where of course persistence is taken into account. If this cannot be done analytically, Monte-Carlo generating methods may be applied.

If we define

$$RS_t^{*\alpha} \text{ by } Prob(\sum_{k=1}^{t} R_k \leq RS_t^{*\alpha}) = \alpha \tag{48}$$

then eq (47) is equivalent to

$$\sum_{k=1}^{t} X_k - S_o + Smin_t \leq RS_t^{*\alpha_1} \tag{49}$$

or

$$\sum_{k=1}^{t} X_k \leq S_o - Smin_t + RS_t^{*\alpha_1} \tag{50}$$

Likewise, eq (46) is replaced by

$$\sum_{k=1}^{t} X_k \geq S_o - Smax_t + RS_t^{*(1-\alpha_2)} \qquad (51)$$

The chance-constrained LP consists of the objective function (eq (42)) and the constraints (eqs (44), (50) and (51)). Curry et al (op cit) actually used more complicated models which explicitly accounted for evaporation and stochastic demand, here only some basic ideas were described. They also extended the models to multireservoir systems, which involved no difficulties, compare ch 5 of this paper. Here both the objective function and the restrictions on release are deterministic, as X is to be fixed beforehand. This means that the stochasticity is transmitted to storage, but this is an approximation in real cases, because if the inflow happens to be very low, it may be impossible to release the predetermined amounts of water, and likewise in case of extremely high inflows we may be forced to increase X.

How deterministic X is in reality depends among others on the limits of S. If Smin corresponds to an empty reservoir, indeed it will sometimes be impossible to maintain the predetermined releases. But if Smin is far above the empty reservoir, we may (if maintaining predetermined releases is considered more important than maintaining storage above Smin) be able to fulfil the scheme of releases. Corresponding facts hold for Smax and high inflows.

The approach to predetermine the releases was called Static Release Policy (SRP) by Luthra-Arora (1976) and it has certain advantages.

Often the objective is a function mainly of release while storage has only secondary influence on the net benefit, consider for example the problem of maximizing delivery of firm water or firm energy. It is then an advantage to know the future releases beforehand.

In the models where release is dependent on storage (as in for example LDR and in the model of Loucks (1969)), releases are random, which means that we cannot optimize the result, only the expected result or some other statistical parameter of the probability distribution of the benefit.

If the active reservoir capacity (which is the difference between Smax and Smin) is small compared to the stochastic variability of inflow, models with predetermined releases are meaningless. In such cases it may be impossible to find any feasible solutions at all to the operation problem if we do not accept to have release dependent on storage and/or inflow. The active storage capacity required and how deterministic X is also depend on the choices of α_1 and α_2. Decreasing the values of α_1 and α_2 increases the required active storage capacity, but it also increases the degree of determinism in X.

The other extreme would be to predetermine the storage, S, and pass all randomness to release. If the purpose is to design a reservoir to be

used for recreation, this approach may be preferable. This approach may be called complete storate regulation while SRP may be called complete outflow regulation. Of course complete storage regulation requires great differences between Xmax and Xmin in the same way as SRP requires large active storage capacity. In most models, however, both storage and releases are random variables.

The drawback with the LDR-approach is that a specific form of release rule is stipulated, while in the approach by Loucks (1969) we have to discretize the variables.

In the SRP-approach no specific form of release rule is stipulated, but as we in all real cases must be ready to change the predetermined releases at least in some cases (for example extremely low inflows), no complete solution to the operation problem is obtained. Moreover, no general release rule is derived, so we have to re-compute the optimal operation for every new situation. An alternative is to apply regression as in ISDP to derive operation rules (see section 2.2). Not even the approach with a fixed form of release rule gives us a complete solution covering all situation. Consider for example the case with high storage and predictions of extremely high inflows to follow. Then indeed we are not probably satisfied by using a release scheme according to the fixed rule, but must re-analyze the operation during this critical time period in order to find the best scheme of releases to prevent flooding. In the SRP-approach persistency is taken into account, but in the LDR-approach it is not. Looking at eq (30) one notices that (Luthra-Arora 1976) in the LDR-approach "the variability inherent in the inflow R_{t-1} is practically passed on to the next periods release X_t, except for the moderation achieved through the addition of the controllable quantity $(b_{i-1}-b_i)$, which stays fixed for any period through the choice of b_i".

Luthra-Arora (1976) combined the SRP and LDR models in the design problem by imagining that a part, $\delta \cdot R_t$, is diverted into a LDR-reservoir while the rest of inflow, $(1-\delta) \cdot R_t$, enters a SRP-reservoir, $0 \leq \delta \leq 1$.

Then the release can be written

$$X_t = \delta \cdot R_{t-1} + b_{i-1} - b_i + Z_t \qquad (52)$$

where Z is the release from the SRP-reservoir, and of course X is the total release from the real reservoir. This equation should be compared with eq (30) which is rewritten below

$$X_t = R_{t-1} + b_{i-1} - b_i \qquad (30)$$

For fixed δ this leads to a LP-problem which may be solved by standard methods. Luthra-Arora varied δ in increments of 0.1 with $0 \leq \delta \leq 1$; for further details we refer to their paper. One rationale of their approach was to distribute the effects of inflow variability between storage and releases.

This chapter ends with a description of a standard method of approximating non-linear programs with linear constraints by Linear Programming.

Consider the NLP

$$\text{Max} \quad Z = \sum_{i=1}^{n} c_i(x_i) \tag{53}$$

subject to

$$\sum_{i=1}^{n} a_{ji} \cdot x_j \begin{matrix} \geq \\ = \\ \leq \end{matrix} b_j \qquad j = 1,2,\ldots,m \tag{54}$$

$$x_i \geq 0 \qquad i=1,2,\ldots,n$$

This differs from the LP in eqs (2)-(4) in that the coefficient (of x_i) in the objective function depends on x_i. Consider Fig 1. Here dc_i/dx_i is a decreasing function of x_i.

The benefit functions are approximated by linear segments as in shown in figure.

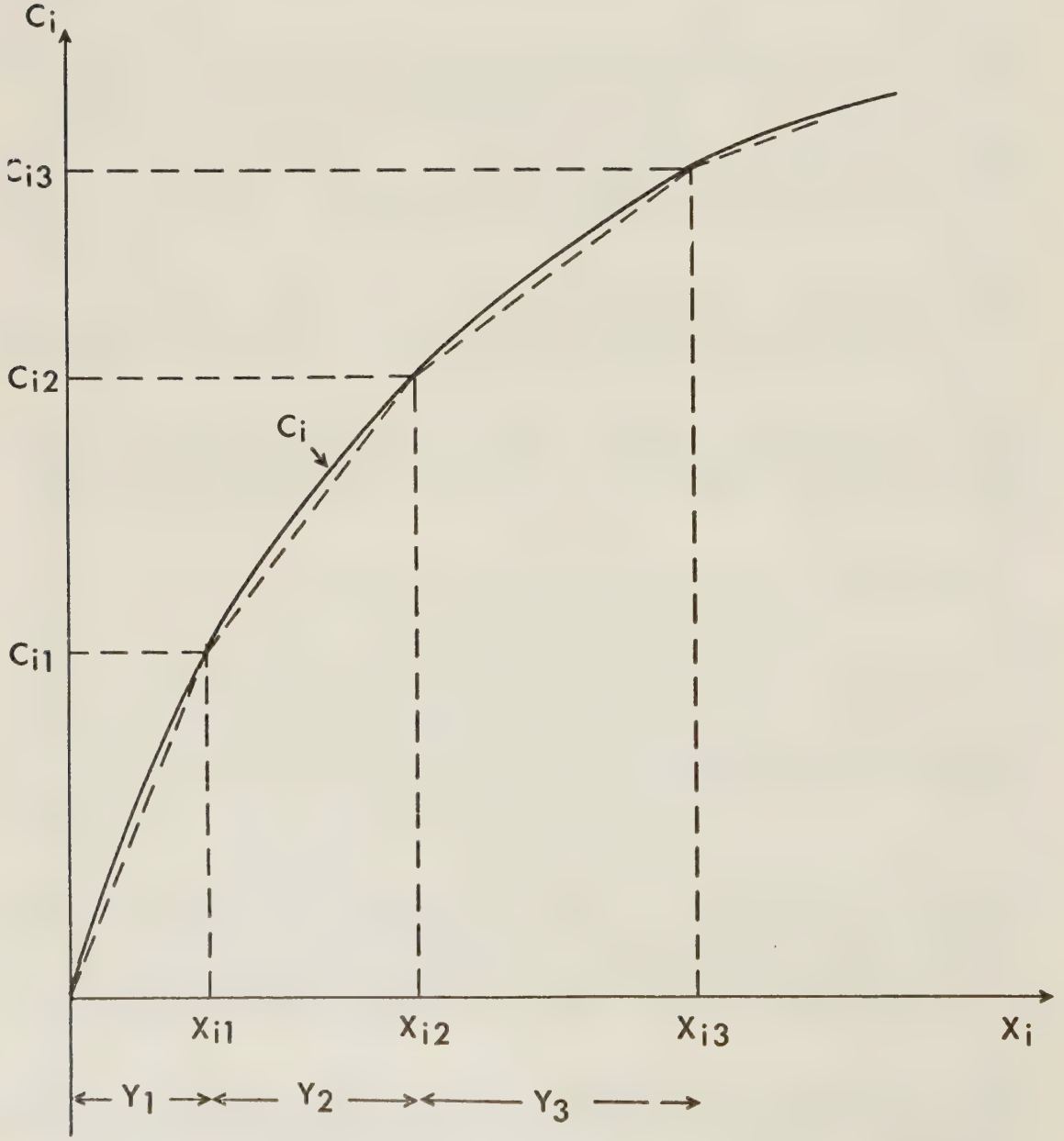

Fig 1. Illustration of a method of approximating a NLP by LP.

Then eq (53) is replaced by

$$\text{Max } ZZ = \sum_{i=1}^{n} \left[\sum_{k=1}^{Kmax_i} d_{ik} \cdot y_{ik} \right] \tag{56}$$

$$d_{il} = c_{il}/x_{il}$$

$$d_{ik}=(c_{ik}-c_{i,(k-1)})/(x_{ik}-x_{i,(k-1)}) \qquad k=2,3,\ldots,kmax_i \tag{57}$$

with

$$x_i = y_{il} + y_{i2} + \ldots + y_{i,kmax_i} \tag{58}$$

This equation is substituted into eq (54) which becomes

$$\sum_{i=1}^{n} a_{ji} \left(\sum_{k=1}^{kmax_i} y_{ik} \right) \begin{matrix} \geq \\ = \\ \leq \end{matrix} b_j \qquad j=1,2,\ldots,m \tag{59}$$

If we for all i (i=1,2,....,n) have $\frac{dc_i}{dx_i}$ decreasing with x_i, the NLP (eqs (53)-(55)) are approximated by eqs (56), (59), (60) and the limits of the y:s (which are bounded between zero and Y_i; see Fig 1).

The fact that the values of additional units of x are decreasing ensures that the y:s are entered in correct order in the solution procedure, that is, we have no risk to obtain for example $y_2 > 0$ and $y_1=0$ in the optimal solution, see for example Wagner (1969). If there exist c:s whoce incremental values increase with x, then additional constraints must be added in order to assure that the y:s are entered in correct order.

3 LARGE SYSTEMS

Application of standard techniques to the optimization of large systems over many time steps is often computationally infeasible, at least considering the limited computer resources that often are available. This has led to development of several special procedures and approximation methods designed to handle large systems. Some applications to water resources are described here, for a general treatment see for example Lasdon (1970) and for water resources Haimes (1977). One way is decomposition, which means that the total system is split up into smaller subsystems, which are easier to analyze and optimize than the total large system. At the lower level the subsystems are analyzed (optimized) and at the higher level a "master" co-ordinates the actions of the subsystems by using their outputs. The outputs may be, for example, a set of releases from reservoirs.

In most versions the optimization proceeds by iteration between the two levels. Usually the master submits "prices" on common resources to the subsystems, which are then optimized individually with respect to these prices. Using the "suggestions" from the subsystems the master sets up new prices, the subsystems are optimized again, and the master submits new prices etc.

If the large original problem is a LP the total optimal solution consists of a weighted average of the suggestions from the subsystems, i.e. the master maintains the decision-making responsibility (Lasdon 1970). But in other cases complete decentralization may be possible, which means that when the master submits appropriate prices to the subsystems, the total optimal solution is obtained. These appropriate prices are obtained during the iteration process.

A "classical" example of decomposition in water resources is the paper by Hall and Shephard (1967). They applied the technique to the northern part of the California water scheme. It is a multipurpose water resources system which is operated for irrigation, hydropower, river navigation etc.

The object of the investigation was to estimate the maximum amount of firm water and firm power which could be generated by the system and to derive an operating policy which would maximize the returns.

The entire system was decomposed into a master and four subsystems, where each of the subsystems consisted of one reservoir and a plant. Prices on power and water were submitted by the master to the subsys-

tems, which were optimized individually by dynamic programming. Using the scheme of releases and power production from the subsystems, the master applied LP to optimize the amounts of firm and nonfirm water and power to be generated by the total system. The dual of this LP yields a set of shadow prices which measure the value of changes in the outputs from the subsystems. These prices are then submitted as new prices to the subsystems, which are optimized using these prices, etc.

Finally the optimal solution is obtained for the total system. In this version complete decentralization was achieved. The approach to uncertain future runoff was to use a critical period with low flows from the historical record.

Butcher and Sundar (1973) decomposed a system of two surface water reservoirs and one groundwater reservoir into two subsystems. The aquifer formed one subsystem and the surface water reservoirs the other one. The aquifer subsystem was optimized conditioned on the operation of the surface water subsystem, and this optimization gave also the derivative of the return function (of the aquifer subsystem) with respect to aquifer storage. This derivative was used in the optimization of the surface water subsystem, and the total optimal operation was obtained by iteration between the two subsystems. The derivative obtained from aquifer optimization may be interpreted as a marginal benefit due to interactive decisions. For further details we refer to their paper. A merit of their decomposition method, is that one of the subsystems (the aquifer) is much "slower" than the other, which made it possible to use different time steps in the optimization procedures applied. This greatly reduced the total computer time for optimization.

For other examples see e g Haimes (1977), where also applications of three-level-optimization are treated.

Another technique used to handle large systems is aggregation, and its application to operation of multireservoir systems is described here. It was applied to a 58-reservoir sewer system by Bradford (1977).

At the highest level the original system of many reservoirs is approximated by a system with fewer reservoirs. This new system is obtained by aggregation, i.e. some reservoirs are combined to single ones by adding their active storage capacities and their inflows. The optimal policy for this simplified system is computed.

At the lower level(s) the system of reservoir aggregates are disaggregated and the policy for smaller aggregates are determined, and at the lowest level the operations of the individual reservoirs are computed.

This method is an approximate method, and we have no guarantee that the optimal solution of the original large system is obtained. Also, the "optimal" solutions obtained at higher levels may be necessary to modify in order to make the lower-level problems feasible (an infeasible problem is a problem which has no feasible solutions). A method of aggregation is applied by the author in chapter 5.

An alternative to ordinary linear programming models, that is sometimes used for multireservoir problems, is the network approach. Other terms used synonymously in operations research are transporation studies and graph theory. In the field of water resources systems network has a somewhat different meaning than in electronics. No definitions or axiomatic treatment is given here, we only discuss the application to the operation of water resources systems.

Consider a river with a number of reservoirs which control the outflows. Certainly the decision to release an amount, X, of water may be thought of as transporting X units of water to the nearest downstream reservoir. By working with discrete time steps and thinking of the system expanded in a new dimension, we may think of storing an amount, S, to the next time period as a transport of S units to a point corresponding to the next time step and the same reservoir. See fig 2 where the network approach to a two-reservoir system is illustrated. Here

Q(I, J) local net inflow to reservoir I during time step J
X(I, J) release from reservoir I during time step J
S(I, J) storage at end of time step J at reservoir I

In this figure each Q, X, and S corresponds to an arc (branch), obviously X and S are adjustable flows while Q is fixed. Unit prices may be put on each arc, as well as lower and upper limits on the flows. For fixed arcs we let the lower and upper limits coincide.

This approach was designed for deterministic inflow, so special techniques must be applied in case of random Q. If the objective function is linear in X and S and all constraints may be expressed as lower and upper limits on the arcs, we have a linear transportation problem. For such problems special algorithms exist which can deal with large problems. As an example we have the Out-of-Kilter algorithm (Durbin-Kroenke 1967).

The network approach to water resources was probably first applied by Texas Water Resources Development Board (1970, 1971 and 1974) who analyzed the Texas Water System by combinations of optimization and simulation studies.

King et al (1971) and Kaczmarek et al (1971) applied the technique to the Vistula Basin in Poland.

In the latter paper, the approach to stochastic inflows was a multistep procedure. In the first step monthly mean inflows were used in the network optimization to define targets of controlled outflow. In the second step stochastic dynamic programming was applied to minimize the expected sum of square deviations from the target releases.

Sigvaldason (1976) applied the Out-of-Kilter algorithm to a 48-reservoir system. He used a simulation approach, which proceeds in a recursive manner from one time step to the next and which involves optimization for each time step. At the start of one time interval the decision maker knows the current reservoir levels and channel flows and uses predictions

of net inflows for the next time interval. With this information the operation is optimized for the next time interval.

A method to use the network approach when having stochastic inflows is described in chapter 5 in this paper.

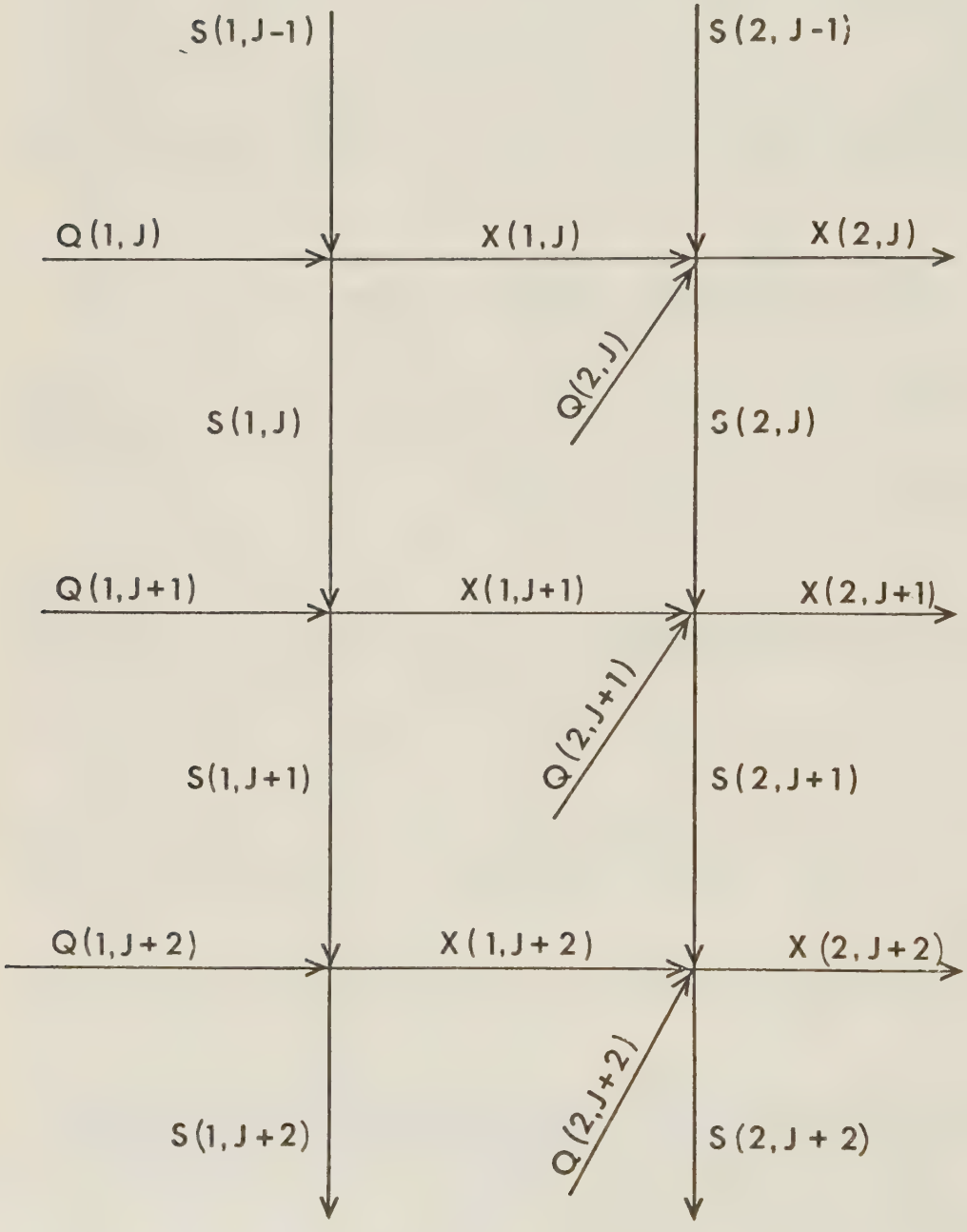

Fig 2. Network representation of a two-reservoir system

4 HYDRO-ELECTRICITY SYSTEMS

Now we turn to the special kind of water resources systems which is the main topic of this paper, reservoirs regulated for hydro-power production.

The operation of such systems is an interdisciplinary field that has traditionally been studied mainly as a branch of electrical engineering. The problem is often divided into two separate problems; the short-term regulation and the long-term regulation problem.

In short-term regulation the time steps may be e g days or hours and the time span is usually less than or equal to one week, while in the long-term regulation problem one week is a usual time step and the time span is a number of weeks, maybe one year.

In the short term problem prediction of inflows is often so successful that they may be considered preknown and deterministic. This is an argument for considering this problem a branch of electrical engineering.

Turning to the long-term regulation, the uncertainty of future inflows is a fundamental feature, and this makes it a problem of stochastic optimization. Moreover some approximations in the electrical engineering domain are often applied; transmission losses are neglected, the different generators within a power plant are approximated by (aggregated into) a single one etc. This means that the science of hydrology has probably more to offer to long-term regulation studies than to short-term ones.

In real cases the hydro-electricity system is combined with thermal plants and maybe also other sources of energy. Generated electricity cannot be stored but must be consumed immediately, and the producers of power are enforced by law to deliver energy according to the demand except for extreme conditions when rationing is used.

The operation problem is to decide how much energy to produce from each of the plants and how much water to release from the different reservoirs.

Little (1955) applied explicit stochastic dynamic programming to the operation of a one-reservoir hydro-power system where the objective was to minimize the expected cost of the supplemental energy production.

McKerchar (1974) applied implicit stochastic dynamic programming to a two-reservoir hydro-power system with the objective of minimizing the

present value of complementary thermal generating costs.

A common method in analysis of multireservoir and multiplant hydro-electricity systems is to aggregate the system into one reservoir, one inflow and one power plant. This approach was applied by Lindqvist (1962) in a study of the long term operation problem. The incremental value of stored water as a function of storage was computed and compared with the incremental cost of thermal power production and the incremental income from selling surplus power. The optimal operation was determined by the criterion that, "during each coming month only as much thermal power is produced (as far as local inflow permits) and only as much surplus power is delivered as corresponds to the incremental value of water".

In operation studies the cost of hydro-power production is (usually) considered equal to zero, but because the water in a reservoir besides being used in the current time step can be saved for power production in future time steps, the units of water stored in a reservoir have an economic value.

The incremental water value depends on the amount of water in storage and this amount is clearly influenced by the inflow. As we usually have persistence in the inflow process, the expected incremental water value for future time steps should be a function of current inflow also.

In Fjeld et al (1973) the merit of utilizing the state of the runoff-process was demonstrated. They applied explicit stochastic dynamic programming to a one-reservoir system.

In a combined optimization-simulation study (Institutt for Vassbygging-Elektrisitetsforsyningens Forskningsinstitutt 1974) not only the expected result but also the variance was investigated. In the optimization procedure the hydro-electricity system was approximated by a single reservoir-single plant system, and by using the incremental water value the optimal operation was derived by ESDP. In the simulation study a stochastic model of inflows was used.

In reality however, the incremental water value for a reservoir depends not only on the storage in this particular dam, but also (among other thing) on the states of other reservoirs. Additionally, each reservoir has its own incremental water value. However, taking these features of the incremental water value into account would lead to a large system with enormous computational requirements.

A "maximum energy storage" approach is sometimes applied. Here the objective is to maximize the hydro-energy storage that remains after having produced the electricity that is required. This approach (in slightly different versions) was applied by Valvis-Mathioudis (1971), Becker-Yeh (1974) and Nesse (1974). It is of limited value for linear models with constant head plants where the power output equals water flow times head, because (unless spilling occurs) all solutions will lead to the same final energy storage.

In Nesse (1974) the approach to unknown future inflows in the long-term regulation problem was to consider the inflow deterministic for the first week of the planning period and stochastic for other weeks. In the optimization step the SRP-approach was used. The risk of shortage of water was treated by putting a price on energy deficits and optimizing the expected result of operation. His long-term studies included a two-reservoir system while larger systems were used in short-term studies.

Dynamic programming was applied by Silver et al (1972) to the operation of a hydro-electricity system explicitly consisting of two reservoirs for the period from December 1 to time of snow-melt inflow. The problem was made deterministic by using long-term predictions of inflow.

5 A COMBINED OPTIMIZATION-SIMULATION APPROACH TO THE OPERATION OF A 5-RESERVOIR HYDRO-ELECTRICITY SYSTEM

5.1 INTRODUCTION

In the river Indalsälven and its tributaries there is a large system of dams constructed and operated for hydro-power production. A small subsystem situated on the tributaries Långan, Korsvattsån and Övre Oldån is shown on the map, Fig 3, and a simplified configuration of the reservoirs Burvattnet, Stora Mjölkvattnet, Korsvattnet, Övre Oldsjön and Landösjön is depicted in Fig 4. This five-reservoir system was chosen as the object for this study on multireservoir stochastic optimization. The following symbols are used:

Q(I,k) local net inflow to reservoir I during time step k
X(I,k) release from reservoir I during time step k
HL the approximate net head for the plant Långså
HO the approximate net head for the plant Oldå

The heads HL and HO are known and functions of storage among other things. But the relative variations caused by storage changes are small, thus we treat HL and HO as constants. The depicted system is only a small part of the total system of the river Indalsälven. The release from Landösjön enters downstream reservoirs and there are several power plants downstream, however none of them are in the immediate vicinity of Landösjön.

In order to optimize this subsystem we have to make some simplifying assumptions concerning the role of the water released from Landösjön into the rest of the system. The approach here is simply to sum all the heads of the power plants downstream from Landösjön. We consider the water released into a single power plant with a head equal to this sum. Time delay is not taken into account, nor are capacity limitations of this fictitious single "plant". The sums of these heads is roughly the same as the net head HO. In principle we should not compare net heads with total heads. However, as an approximation of the interaction between our subsystem and the rest of the river-system, we can ignore the difference. Thus

$$HLA = HO$$

HLA head of the "plant" downstream from Landösjön.

Moreover, this is just an optimization and simulation study applied to a simplified system. No attempts are made to cover, for example, the complete set of constraints (legal and technical), or the detailed electrical engineering and economic realities of the real system.

5.2 OPTIMIZATION MODEL

We consider production of electricity according to a curve of relative demand, the time horizon equals a number of weeks and the time unit is one week.

P_k	Power produced during time step k
B_k	Relative demand in time step k
PN	"Normal" electricity production (a parameter to be optimized) by our system
NT	Number of time steps in the planning period

That is, we want to have

$$P_k = B_k \cdot PN \tag{61}$$

$$HL \cdot X(2,k)+HO \cdot X(4,k)+HLA \cdot X(5,k)= P_k \tag{62}$$

$$(K=1,2,3,\ldots,NT)$$

The energy is expressed in units of water released times head in the equations above.

As the object of the optimization a natural choice is:

$$\text{Max PN} \tag{63}$$

Turning to the constraints, we have of course equations (61) and (62). In the problem formulation used here, we determine the releases at the start of our time span, (compare discussion in section 2.4). Obviously the total amount of water released must not be greater than allowing a minimum storage to be maintained.

$$\sum_{k=1}^{T} X(1,k) \leq S(1,0)+ \sum_{k=1}^{T} Q(1,k)-Smin(1,T) \tag{64}$$

Smin(I,k)	Minimum storage required at reservoir I at the end of time step k
S(I,0)	Initial storage at reservoir I

On the right side the second term is accumulated local net inflow, and the left side is accumulated release.

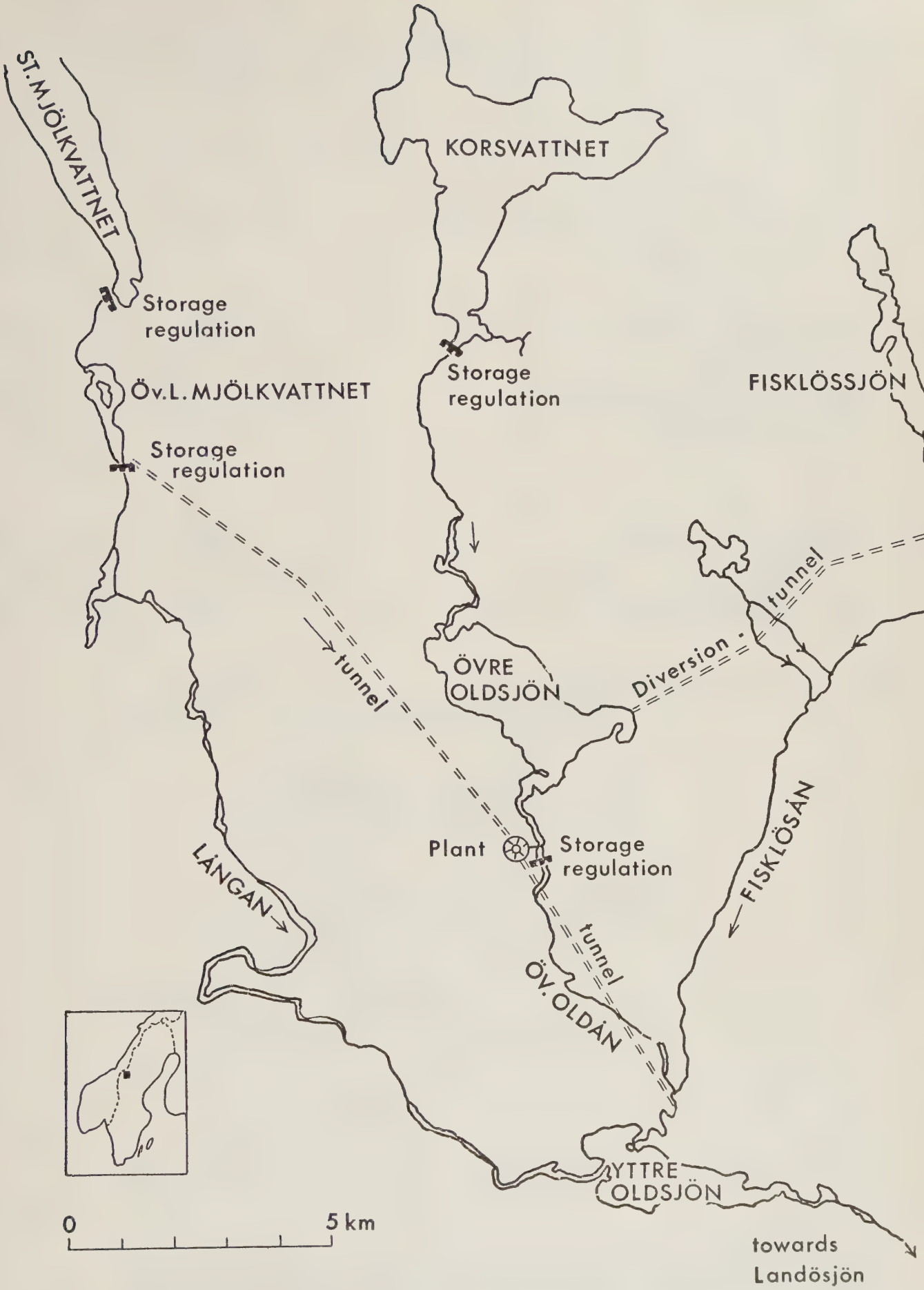

Fig 3. Part of the reservoir system

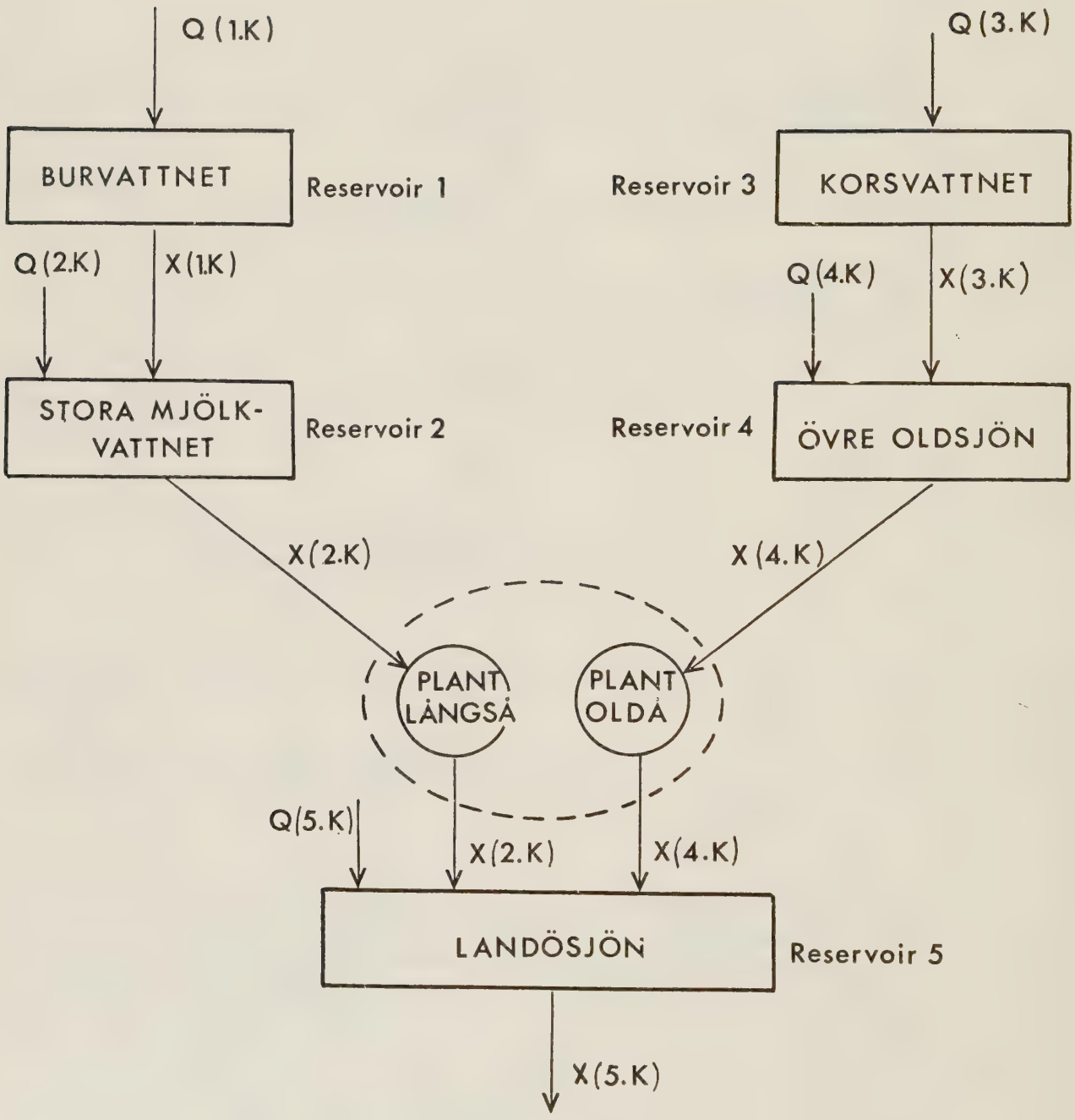

Fig 4. Configuration of the simplified system used in this report

$$\text{Prob}\left\{\sum_{k=1}^{T} Q(I,k) \geq QS^{*\,(1-\beta)}_{(I,T)}\right\} = \beta$$

and (70)-(74) are replaced by

$$\sum_{k=1}^{T} X(1,k) \geq S(1,0)-Smax(1,T)+QS^{*\,(1-\beta)}_{(I,T)} \quad (80)$$

$$\sum_{k=1}^{T} (-X(1,k)+X(2,k)) \geq S(2,0)-Smax(2,T)+QS^{*\,(1-\beta)}_{(2,T)} \quad (81)$$

$$\sum_{k=1}^{T} X(3,k) \geq S(3,0)-Smax(3,T)+QS^{*\,(1-\beta)}_{(3,T)} \quad (82)$$

$$\sum_{k=1}^{T} (-X(3,k)+X(4,k)) \geq S(4,0)-Smax(4,T)+QS^{*\,(1-\beta)}_{(4,T)} \quad (83)$$

$$\sum_{k=1}^{T} (-X(2,k)-X(4,k)+X(4,k)) \geq S(5,0)-Smax(5,T)+QS^{*\,(1-\beta)}_{(5,T)} \quad (84)$$

Returning to the deterministic version, as is well known we can replace the constraints corresponding to eqs (64)-(74) with

$$S(1,k-1)+Q(1,k)-X(1,k) = S(1,k) \quad (85)$$

$$S(2,k-1)+X(1,k)+Q(2,k)-X(2,k) = S(2,k) \quad (86)$$

$$S(3,k-1)+Q(3,k)-X(3,k) = S(3,k) \quad (87)$$

$$S(4,k-1)+X(3,k)+Q(4,k)-X(4,k) = S(4,k) \quad (88)$$

$$S(5,k-1)+X(4,k)+X(2,k)+Q(5,k)-X(5,k) = S(5,k) \quad (89)$$

(the continuity equations) and

$$Smin(I,k) \leq S(I,k) \leq Smax(I,k) \quad (90)$$

and as before we of course must have

$$Xmin(I,k) \leq X(I,k) \leq Xmax(I,k) \quad (91)$$

In the equations above $k=1,2,3,\ldots,NT$ and $I=1,2,3,4,5$. $S(I,k)$ is the storage at reservoir I at end of time step k.

The equations above includes all constraints of our deterministic version except the production constraint (eqs (61) and (62)). This shows that the reduced problem (neglecting the production constraint) has a network structure (compare Fig 2).

However, in the stochastic version this transformation is not possible, because $QS^{*\alpha}(I,T)$ is not in general equal to $QS^{*(1-\beta)}(I,T)$. Let us define

$$Q^{**\gamma}(I,1) = QS^{*\gamma}(I,1) \tag{92}$$

$$Q^{**\gamma}(I,k) = QS^{*\gamma}(I,k) - QS^{*\gamma}(I,k-1) \tag{93}$$

where

$I=1,2,3,4,5.$

$k=2,3,4,\ldots\ldots,NT$

$0 \leq \gamma \leq 1$

Then

$$QS^{*\gamma}(I,T) = \sum_{k=1}^{T} Q^{**\gamma}(I,k) \tag{94}$$

Substituting these relations into eqs (75)-(79) we obtain

$$\sum_{k=1}^{T} X(1,k) \leq S(1,0) + \sum_{k=1}^{T} Q^{**\alpha}(1,k) - Smin(1,T) \tag{95}$$

$$\sum_{k=1}^{T} (-X(1,k)+X(2,k)) \leq S(2,0) + \sum_{k=1}^{T} Q^{**\alpha}(2,k) - Smin(2,T) \tag{96}$$

$$\sum_{k=1}^{T} X(3,k) \leq S(3,0) + \sum_{k=1}^{T} Q^{**\alpha}(3,k) - Smin(3,T) \tag{97}$$

$$\sum_{k=1}^{T} (-X(3,k)+X(4,k)) \leq S(4,0) + \sum_{k=1}^{T} Q^{**\alpha}(4,k) - Smin(4,T) \tag{98}$$

$$\sum_{k=1}^{T} (-X(2,k)-X(4,k)+X(5,k)) \leq S(5,0) + \sum_{k=1}^{T} Q^{**\alpha}(5,k) - Smin(5,T) \tag{99}$$

In the same way, eqs (80)-(84) are rewritten

$$\sum_{k=1}^{T} X(1,k) \geq S(1,0) - Smax(1,T) + \sum_{k=1}^{T} Q^{**(1-\beta)}(1,k) \tag{100}$$

$$\sum_{k=1}^{T} (-X(1,k)+X(2,k)) \geq S(2,0) - Smax(2,T) + \sum_{k=1}^{T} Q^{**(1-\beta)}(2,k) \tag{101}$$

$$\text{Prob}\left\{\sum_{k=1}^{T} Q(I,k) \geq QS^{*\,(1-\beta)}_{(I,T)}\right\} = \beta$$

and (70)-(74) are replaced by

$$\sum_{k=1}^{T} X(1,k) \geq S(1,0)-Smax(1,T)+QS^{*\,(1-\beta)}_{(I,T)} \quad (80)$$

$$\sum_{k=1}^{T} (-X(1,k)+X(2,k)) \geq S(2,0)-Smax(2,T)+QS^{*\,(1-\beta)}_{(2,T)} \quad (81)$$

$$\sum_{k=1}^{T} X(3,k) \geq S(3,0)-Smax(3,T)+QS^{*\,(1-\beta)}_{(3,T)} \quad (82)$$

$$\sum_{k=1}^{T} (-X(3,k)+X(4,k)) \geq S(4,0)-Smax(4,T)+QS^{*\,(1-\beta)}_{(4,T)} \quad (83)$$

$$\sum_{k=1}^{T} (-X(2,k)-X(4,k)+X(4,k)) \geq S(5,0)-Smax(5,T)+QS^{*\,(1-\beta)}_{(5,T)} \quad (84)$$

Returning to the deterministic version, as is well known we can replace the constraints corresponding to eqs (64)-(74) with

$$S(1,k-1)+Q(1,k)-X(1,k) = S(1,k) \quad (85)$$

$$S(2,k-1)+X(1,k)+Q(2,k)-X(2,k) = S(2,k) \quad (86)$$

$$S(3,k-1)+Q(3,k)-X(3,k) = S(3,k) \quad (87)$$

$$S(4,k-1)+X(3,k)+Q(4,k)-X(4,k) = S(4,k) \quad (88)$$

$$S(5,k-1)+X(4,k)+X(2,k)+Q(5,k)-X(5,k) = S(5,k) \quad (89)$$

(the continuity equations) and

$$Smin(I,k) \leq S(I,k) \leq Smax(I,k) \quad (90)$$

and as before we of course must have

$$Xmin(I,k) \leq X(I,k) \leq Xmax(I,k) \quad (91)$$

In the equations above $k=1,2,3,\ldots,NT$ and $I=1,2,3,4,5$. $S(I,k)$ is the storage at reservoir I at end of time step k.

The equations above includes all constraints of our deterministic version except the production constraint (eqs (61) and (62)). This shows that the reduced problem (neglecting the production constraint) has a network structure (compare Fig 2).

However, in the stochastic version this transformation is not possible, because $QS^{*\alpha}(I,T)$ is not in general equal to $QS^{*(1-\beta)}(I,T)$. Let us define

$$Q^{**\gamma}(I,1) = QS^{*\gamma}(I,1) \tag{92}$$

$$Q^{**\gamma}(I,k) = QS^{*\gamma}(I,k) - QS^{*\gamma}(I,k-1) \tag{93}$$

where

$I=1,2,3,4,5.$

$k=2,3,4,\ldots\ldots,NT$

$0 \leq \gamma \leq 1$

Then

$$QS^{*\gamma}(I,T) = \sum_{k=1}^{T} Q^{**\gamma}(I,k) \tag{94}$$

Substituting these relations into eqs (75)-(79) we obtain

$$\sum_{k=1}^{T} X(1,k) \leq S(1,0) + \sum_{k=1}^{T} Q^{**\alpha}(1,k) - Smin(1,T) \tag{95}$$

$$\sum_{k=1}^{T} (-X(1,k)+X(2,k)) \leq S(2,0) + \sum_{k=1}^{T} Q^{**\alpha}(2,k) - Smin(2,T) \tag{96}$$

$$\sum_{k=1}^{T} X(3,k) \leq S(3,0) + \sum_{k=1}^{T} Q^{**\alpha}(3,k) - Smin(3,T) \tag{97}$$

$$\sum_{k=1}^{T} (-X(3,k)+X(4,k)) \leq S(4,0) + \sum_{k=1}^{T} Q^{**\alpha}(4,k) - Smin(4,T) \tag{98}$$

$$\sum_{k=1}^{T} (-X(2,k)-X(4,k)+X(5,k)) \leq S(5,0) + \sum_{k=1}^{T} Q^{**\alpha}(5,k) - Smin(5,T) \tag{99}$$

In the same way, eqs (80)-(84) are rewritten

$$\sum_{k=1}^{T} X(1,k) \geq S(1,0) - Smax(1,T) + \sum_{k=1}^{T} Q^{**(1-\beta)}(1,k) \tag{100}$$

$$\sum_{k=1}^{T} (-X(1,k)+X(2,k)) \geq S(2,0) - Smax(2,T) + \sum_{k=1}^{T} Q^{**(1-\beta)}(2,k) \tag{101}$$

$$\sum_{k=1}^{T} X(3,k) \geq S(3,0)-Smax(3,T)+\sum_{k=1}^{T} Q^{**(1-\beta)}(3,k) \qquad (102)$$

$$\sum_{k=1}^{T} (-X(3,k)+X(4,k)) \geq S(4,0)-Smax(4,T)+\sum_{k=1}^{T} Q^{**(1-\beta)}(4,k) \qquad (103)$$

$$\sum_{k=1}^{T} (-X(2,k)-X(4,k)+X(5,k)) \geq S(5,0)-Smax(5,T)+\sum_{k=1}^{T} Q^{**(1-\beta)}(5,k) \qquad (104)$$

Comparing eqs (64)-(68) with (95)-(99) we see that they are of quite the same form, but with Q replaced by $Q^{**\alpha}$. The same hold for eqs (70)-(74) and (100)-(104) where Q is replaced by $Q^{**(1-\beta)}$. If we consider constraints against shortage only, we could look upon $Q^{**\alpha}$ as a fictitious known "inflow". Correspondingly, if emphasizing constraints against overflow and spillage, $Q^{**(1-\beta)}$ is the known "inflow". Thus if we want to take into account both shortage and overflow, in the stochastic version, we have to find a set of releases X, that satisfies two sets of "inflows", $Q^{**\alpha}$ and $Q^{**(1-\beta)}$, while in the deterministic case we have just one Q-matrix. (In both cases X must satisfy equations (61) and (62).) Or equivalently, we have to find a matrix of releases X, that could be routed through two networks, equivalent in all aspects except Q.

It should be emphasized that $Q^{**\gamma}$ is in general quite different from $Q^{*\gamma}$ defined by

$$\text{Prob}\left\{ Q(I,T) \leq Q^{*\gamma}(I,T) \right\} = \gamma$$

i e the 100γ: -percentile of Q(I, T).

The solution method applied to our optimization model is described in section 5.4.

Persistence is obviously taken into account in our approach, but what about the space-dependence, i e the inter-station dependence?

Looking at equations (95)-(104) one notices that we consider the risks for each reservoir separately. In reality, however, a low inflow to one of our reservoirs is usually accompanied by low inflows to the other ones, the same concerns high and medium flows. An interesting question is the probability of, for example, shortage at at least one of our 5 reservoirs.

We discuss the case when we have SL=Smin at all our dams simultaneously.

Let us look at two extreme cases:

PSl Probability of shortage at at least one dam

Case A: No inter-station dependence in the inflows.
Then we have

PSl=1-Prob (no shortage at any of our 5 dams)

$$PSl=1-(1-\alpha)^5$$

Case B: Perfect correlation between the dams.
Then shortage at one dam occurs if and only if shortage occurs at the other dams also, i e

$$PS1=\alpha$$

PS1 decreases when the inter-station dependence increases. On the other hand, the lower this dependence, the greater are the possibilities of obtaining high inflow at another reservoir as compensation for the low inflow to one particular dam. In our case we have neither of the extreme cases A or B, but are closer to case B than case A.

5.3 INPUT DATA TO THE OPERATION STUDIES

Our historical inflow series are 1945-75 for Stora Mjölkvattnet, 1945-73 for Korsvattnet and Landösjön. Inflow data also exists for Burvattnet, but instead of using this record, inflow to this reservoir is proportionated from Stora Mjölkvattnet, this is considered to give at least the same confidence as using the record from Burvattnet. Inflow to Övre Oldsjön is proportionated from Korsvattnet. We apply

$$Q(1,k) = (142.7/190.9) \cdot Q(2,k) \quad (105)$$

$$Q(4,k) = 1.256 \cdot Q(3,k) \quad (106)$$

142.7 and 190.9 are the mean annual inflow to Burvattnet and Stora Mjölkvattnet respectively expressed in Mm^3 for 1945-67. The number 1.256 is obtained in a similar way.

Returning to Fig 3, we notice the reservoir Öv L Mjölkvattnet. Its storage capacity is too small to have any influence on weekly regulation, but it is stipulated by law, that the runoff in the downstream natural river bed must not be less than 1 m^3/s after the start of the spring-flow. That part of the local inflow into Öv L Mjölkvattnet, which is not necessary for maintaining this minimum, should pass through the power-plant. The mean of the local inflow between Stora Mjölkvattnet and the tunnel is about 2 m^3/s. In general, a little more than the release X(2,k) is thus used in the Långså-plant, but sometimes even less than this amount. To simplify the problem the following approximations were made:

The local inflow below Stora Mjölkvattnet flows directly into Landösjön and is added to its local inflow, the power produced by part of it in the Långså-plant is not taken into account. All of X(2,k) is assumed to pass through the Långså-plant. As regards the lake Fisklössjön, the diversion tunnel automatically brings water to Övre Oldsjön, and this amount of

water is included in its local inflow.

There are some small dams below the Långså and Oldå plants upstream from Landösjön. Their influences on weekly regulations are very small, however, and they are not taken into account.

QTV local inflow between Stora Mjölkvattnet and the Långså plant.

This quantity is estimated by the simple relation

$$QTV = 0.349 \cdot Q \text{ (Stora Mjölkvattnet)}$$

In Table 1 below is shown the drainage areas, storage capacities and mean annual inflows for Burvattnet, Stora Mjölkvattnet, Korsvattnet, Övre Oldsjön and Landösjön. Here QTV and its area are included in the values for Landösjön; the same concerns Fisklössjön and Övre Oldsjön.

1 VE = the amount of water corresponding to 1 m^3/s during one week.

Table 1 Drainage areas, storage capacities and mean annual inflows. Drainage areas and local inflows are local quantities, viz in the drainage area of Mjölkvattnet the catchment of Burvattnet is not included, etc.

Reservoir	Drainage area	Storage capacity	Mean annual inflow (1945-67)	
			VE	$l/(s \cdot km^2)$
Burvattnet	117 km^2	99.3 VE	235.9	38.7
Mjölkvattnet	147	268.3	268.3	41.2
Korsvattnet	85	90.0	176.4	39.8
Övre Oldsjön	120	204.6	221.6	35.4
Landösjön	957	269.7	773.5	15.5

The heads of the plants are

HL = 191 m
HO = 249 m
HLA = 249 m ("Fictitious plant", see section 5.1.)

The relative demand, B_k, which is used in this study is shown in Table 2. The sum of all B_k equals 52.15, which corresponds to the number of weeks in a non-leap year.

Smin = 0 was used except for the last time step in the planning period, see section 5.5, and for Smax the storage capacities were used.

Xmax for Stora Mjölkvattnet and Övre Oldsjön were put equal to the plant capacities, because even if it is possible to spill, indeed it is desirable to use all the water in the turbines.

Table 2 The relative demand, B_k, used in this report

Week	B_k	Week	B_k
1	0.66	27	0.87
2	0.67	28	0.71
3	1.25	29	0.71
4	1.23	30	0.77
5	1.22	31	0.86
6	1.21	32	0.95
7	1.19	33	1.00
8	1.18	34	1.03
9	1.17	35	1.03
10	1.16	36	1.06
11	1.15	37	1.07
12	1.14	38	1.09
13	0.98	39	1.10
14	0.96	40	1.11
15	0.95	41	1.12
16	0.95	42	1.13
17	0.95	43	1.15
18	0.80	44	0.96
19	0.91	45	0.99
20	0.90	46	1.21
21	0.89	47	1.23
22	0.89	48	1.24
23	0.89	49	1.25
24	1.01	50	1.26
25	0.82	51	0.66
26	0.80	52	0.66

The values of Xmax were 20.0, 34.0, 20.0, 31.0, and 200.0 VE for reservoirs 1,2,3,4 and 5 respectively.

For Xmin we used (unit: VE)

Burvattnet:
1.3 week 22
2.0 weeks 23-39
0.7 rest of the year

Stora Mjölkvattnet: 1.2

Korsvattnet:
1.0 week 22
1.5 weeks 23-39
0.6 rest of the year

Övre Oldsjön 0.0

Landösjön:
8.7 week 22
15.0 weeks 23-39
4.4 rest of the year

Xmax for Korsvattnet, Burvattnet and Landösjön were chosen somewhat arbitrarily. Of course the technical discharge capacities depend on storage, and we chose values corresponding to rather large storage. In most real situations, large releases will be made only in time of rather large storage; nevertheless, the computations made never resulted in releases being in the neighbourhood of Xmax for Burvattnet, Korsvattnet or Landösjön.

Xmin was put equal to the minimum release required by law. When computing $QS^{*\alpha}$ and $QS^{*(1-\beta)}$ we could use either the historical inflow series or stochastic models. Our historical series are rather short (about 30 years), which means that (as is common) the estimation of for example $QS^{*0.10}$ and $QS^{*0.90}$ is a non-trivial problem. The historical series is too short to enable good estimates of such quantities direct from these records. But 30-year records may be enough to build stochastic models and estimate the model-parameters (compare Willén (1979)).

Anyhow, stochastic models of inflows to Stora Mjölkvattnet and Korsvattnet were developed (Willén 1979) and these models are used in computing $QS^{*\alpha}$ and $QS^{*(1-\beta)}$ for these reservoirs (and by proportionating for Burvattnet and Övre Oldsjön). A detailed description of these models and also of these historical inflow series is contained in Willén (1979), only a short description is given here.

Owing to snow accumulation, inflow usually is minimum during winter, reaches its maximum during spring because of snow-melt, decreases during summer, and sometimes increases during autumn. Turning to the stochastic models, they actually consist of two parts, one for the "spring", viz the weeks nos 15-26, and another type for the rest of the year.

The spring model is divided into two steps:

Step 1. Generate the total amount of inflow during the spring (the spring-flow) according to its probability distribution.

Step 2. Distribute this amount over the weeks according to the stochastic model.

In step 1 the parameters of the probability distribution depend on winter inflow see Willén (1979).

Since the spring-flow largely consists of snow-melt, the two-step procedure approximates that of nature.

Turning to the other parts of the year, we use conditional probability distributions of the form:

$$F(QQ \mid I) = \text{Prob}\left\{ \begin{array}{l|l} \text{Inflow of next week} \leqslant QQ & \text{Present inflow within the range defined by class I} \end{array} \right\}$$

These conditional distributions are, of course, not the same for the entire non-spring period.

Initially we also used a stochastic model for Landösjön, of the same kind, but with the spring-season weeks 13-24. But because of the fair quality of data of this inflow series, it is not considered possible to enable reasonable success in stochastic model building and parameter estimation. The historic series is too short, it is true, but for this reservoir probably no better method exists than estimation direct from this series. Except for the somewhat earlier snow-melt the inflow series of Landösjön shows similar features to those of the other stations used in this report.

When computing the percentiles of

$$\sum_{k=1}^{T} Q(I,k)$$

by the stochastic models, we generate 1000 series by "random" numbers (the generation procedure is described in Willén 1979), compute the sums and range them.

To start these generations we must know the initial state of the stochastic process. This state is defined by last inflow for the non-spring period, by remaining sum of spring inflow and last inflow for the spring period.

In the computations made for this study different times of the year were used as starting points for the generations, but only one of them (week 18) was in the spring period. In the initial version (where we used a stochastic model of Landösjön also) we assumed perfect knowledge of the remaining sum of spring inflow for the first year. We generated only 250 series for each reservoir instead of 1000.

Later the assumption of perfect predictability of this sum was abolished and also these sums were generated by random methods even when starting at week 18. This was done by applying the equation

$$QV_{rest} = a+b\cdot QH+c\cdot (Q_{15} + Q_{16} + Q_{17})+S \cdot \epsilon \qquad (108)$$

for Korsvattnet, and a similar equation was used for Stora Mjölkvattnet but with QH replaced with ln(QH).

QV_{rest} Generated remaining inflow during spring period

a, b, c Coefficients in the multiple regression

Q_t Inflow during week t

ϵ Independent normally distributed random number with zero mean and unit standard deviation

QH Sum of inflow during some weeks in the winter, weeks 47 last year to week 14 (inclusive) present year for Korsvattnet, and weeks 51 last year to week 14 (inclusive) present year for Stora Mjölkvattnet

S Standard deviation of the residuals (S . ϵ)

The coefficients of the regressions were obtained from the historical series and we obtained

Table 3. Results from the multiple regression in eq (108)

Reservoir	a	b	c	r	S
Korsvattnet	66.3150	1.0071	1.4683	0.6972	12.62
Stora Mjölkvattnet	-10.1525	50.2541	-0.1785	0.7027	22.85

Here r is the correlation between QV_{rest} and $\hat{QV}_{rest}$ computed from eq (108) with $\epsilon = 0$.

The results of the Monte-Carlo generations are probability distributions of

$$QS(I,T) = \sum_{k=1}^{T} Q(I,k)$$

$$(I = 1,2,3,4,5), \quad (T = 1,2,\ldots,NT)$$

These distributions are the forecasts of sums of future inflows, performed by our stochastic models. Some results from our forecasts with our new models (1000 series generated for each forecast) are shown in appendix 2. In the operation model we only use $QS * 0.10$ and $QS * 0.90$, but in the appendix $QS * 0.50$ is also included, and for comparison the QS that really occurred is also plotted.

5.4 SOLUTION METHOD TO THE OPTIMIZATION MODEL

Our problem is defined by the objective function (eq (63)) and by the constraints expressed for example by eqs (61), (62), (69) and (95) - (104). Combining eqs (61) and (62) and summing over the time steps we have

$$PN \sum_{k=1}^{T} B_k = HL \cdot \sum_{k=1}^{T} X(2,k) + HO \cdot \sum_{k=1}^{T} X(4,k) + HLA \cdot \sum_{k=1}^{T} X(5,k) \qquad (109)$$

Eqs (75) + (76), (77) + (78) and (75) + (76) + (77) + (78) + (79) giving respectively

$$\sum_{k=1}^{T} X(2,k) \leqslant \sum_{I=1}^{2} (S(I,0) + QS^{*\alpha}(I,T) - Smin(I,T)) \qquad (110)$$

$$\sum_{k=1}^{T} X(4,k) \leqslant \sum_{I=3}^{4} (S(I,0) + QS^{*\alpha}(I,T) - Smin(I,T)) \qquad (111)$$

$$\sum_{k=1}^{T} X(5,k) \leq \sum_{I=1}^{5} (S(I,0)+QS \quad (I,T)-Smin(I,T)) \tag{112}$$

The right hand side expressions could be interpreted as total water resources for the Långså-plant, the Oldå-plant and the fictitious plant respectively. Inserting eqs (110), (111) and (112) into eq (109) we obtain

$$\begin{aligned} PN \sum_{k=1}^{T} B_k \leq HL \sum_{I=1}^{2} (S(I,0)+QS \quad (I,T)-Smin(I,T)) + \\ HO \sum_{I=3}^{4} (S(I,0)+QS \quad (I,T)-Smin(I,T) + \\ HLA \sum_{I=1}^{5} (S(I,0)+QS \quad (I,T)-Smin(I,T)) \end{aligned} \tag{113}$$

Since these relationships should hold for all values of T (T=1,2,3,.....,NT), and our objective is the maximum PN, we may try

$$PN1=\underset{1 \leq T \leq NT}{Min} \; (RHS(T)/(\sum_{k=1}^{T} B_k)) \tag{114}$$

where RHS is the right hand side of eq (113).

It should be noticed that eq (114) is derived utilizing eqs (110) - (112) which are weaker restrictions than the original ones eqs (76), (78) and (79). Eq (110), for example, treats reservoirs 1 and 2 as one single dam. This means that our method may be considered a variant of aggregation (compare chapter 3). It may happen, that we find a matrix X satisfying eqs (61) and (62) with PN = PN1 and satisfying eqs (110) - (112), but that it is impossible to find a matrix X that besides these restrictions also satisfies e.g. eqs (75), (76) and (69). The storage capacities of single reservoirs, the restricted range of permissible releases, etc, may be limiting factors.

We also have to consider constraints against overflow and spillage, eqs (80) - (84). Making the same kinds of simplifications as made when deriving eqs (110) - (112) we write

$$\sum_{k=1}^{T} X(2,k) \geq \sum_{I=1}^{2} (S(I,0)+QS^{*(1-\beta)} \quad (I,T)-Smax(I,T)) \tag{115}$$

$$\sum_{k=1}^{T} X(4,k) \geq \sum_{I=3}^{4} (S(I,0)+QS^{*(1-\beta)} \quad (I,T)-Smax(I,T)) \tag{116}$$

$$\sum_{k=1}^{T} X(5,k) \geq \sum_{I=1}^{5} (S(I,0)+QS^{*(1-\beta)} \quad (I,T)-Smax(I,T)) \tag{117}$$

Using these relations in eq (109), we obtain

$$PN \geq$$

$$\left(HL \sum_{I=1}^{2} (S(I,0)+QS^{*(1-\beta)}\right) (I,T)-Smax(I,T)) +$$

$$HO \sum_{I=3}^{4} (S(I,0)+QS^{*(1-\beta)}) (I,T)-Smax(I,T)) + \qquad (118)$$

$$HLA \sum_{I,1}^{5} (S(I,0)+QS^{*(1-\beta)}) (I,T)-Smax(I,T)/(\sum_{k=1}^{T} B_k))$$

As this must hold for all T (T = 1,2,3,.....,NT), the minimum possible PN to use considering overflow and spillage is (denoting the right hand side of eq (118) as RHS2):

$$PN2 = \underset{1 \leq T \leq NT}{\text{Max}} (RHS2(T)) \qquad (119)$$

Now, if PN2 > PN1, we have no feasible solution to our problem. Even if PN2 ≤ PN1 it is not quite certain that a feasible solution exists - see comments concerning PN1.

Assuming that PN2 ≤ PN1 it is reasonable to look for a matrix X satisfying PN=PN1 and the constraints. When PN1 ≥ PN2 power production according to the relative demand may be possible, i.e. in a sense "full regulation". Section 5.2 treats this as a problem to find a matrix X satisfying the two matrices of inflow, $Q^{**\alpha}$ and $Q^{*(1-\beta)}$. We define a starting solution by

$$X(I,k) = B_k \cdot RS(I,TT)/(\sum_{j=1}^{TT} B_j) \qquad (120)$$

where RS(I,TT) = the right hand side of eqs (75), (110), (77), (111), and (112) for I=1,2,3,4,5 respectively.

TT The particular value of T that gives PN1 in eq (114).

This choice of X ensures a power production following the curve of relative demand, and that eq (113) will hold as equality at the critical time step TT. In the computations made in this study last time step corresponds to week 17, which is in the spring period but in the early stages of snow-melt in ordinary years. For this week we always obtained TT=NT.

We define

SH = Storage obtained if $Q=Q^{**(1-\beta)}$

SL = Storage obtained if $Q=Q^{**\alpha}$

Of course, SH and SL depend on X, see Fig 5. In fact we have

$$\text{Prob}\left\{ S(I,k) > SH(I,k) \right\} = \beta$$
$$\text{Prob}\left\{ S(I,k) < SL(I,k) \right\} = \alpha \qquad (121)$$

Turning to network terminology, we look upon SH, SL and X as adjustable flows, while Q and S(I, 0) are fixed flows. Smax, Smin, Xmax and Xmin are branch (arc) capacities. Suppose then that the X at hand results in one or more infeasible branches (in at least one of our two networks). Then we choose one of these arcs and try to bring it into feasibility - without driving any other arc into infeasibility or making it more infeasible in any of or our two networks. We continue this procedure until all arcs are feasible, if this is possible. Fig 5 shows a simple example, where we suppose that SH(1,7) > Smax(1,7). Thus, we want to decrease the flow in this arc by the amount

$$DEL=SH(1,7)-Smax(1,7)$$

In order to maintain continuity, we have to find a "cycle" in the networks in which to route DEL, such that no new infeasibility is created in any of our two networks. Fig 5 shows such a cycle, meaning that SH(1,6), SH(1,7) X(1,8), SL(1,6), SL(1,7) are decreased with DEL, while X(1,6), SH(2,6), SH(2,7), SL(2,6) and SL(2,7) are increased with DEL. These relationships should only illustrate the procedure. It is not stated that this particular cycle is performed in any of the computation made for this study. It is possible that we find a route in which we can circulate only a fraction of DEL. We route this fraction, and try to find another cycle by which to route the rest of DEL. The attempts to find a feasible X are made with a method similar to that used in the Out-of-Kilter algorithm, applied in, for example, transportation problems (Durbin and Kroenke 1967). However, the method has to be modified when we deal with arcs corresponding to X(2,k), X(4,k) or X(5,k) owing to the production constraint (eqs (61) and (62)). When such arcs are dealt with, one or more new circulations have to be made in order to maintain power production according to the curve of relative demand.

5.5 RESULTS AND COMBINATIONS WITH A SIMPLE SIMULATION MODEL

Initially we tried one optimization for a whole year, starting with k=1 at week 18 with assumed known conditions at end of week 17 (state of stochastic inflow processes and storages in the reservoirs) and ending at week 17 next year (year 2), i.e. NT=52. As week 17 is in the early stages of snow-melt in ordinary years, it is not surprising that we obtained TT=NT in all computations, i.e. the last time step is the critical one considering shortage. This means that storage is usually low at end of time step NT, so if no lower limits are put on Smin(I,NT) (I=1,2,3,4,5), the risk of empty reservoirs at end of the planning period is considerable. Thus non-zero Smin (I,NT) were chosen in order to have a reserve in case of late snow-melt in year 2.

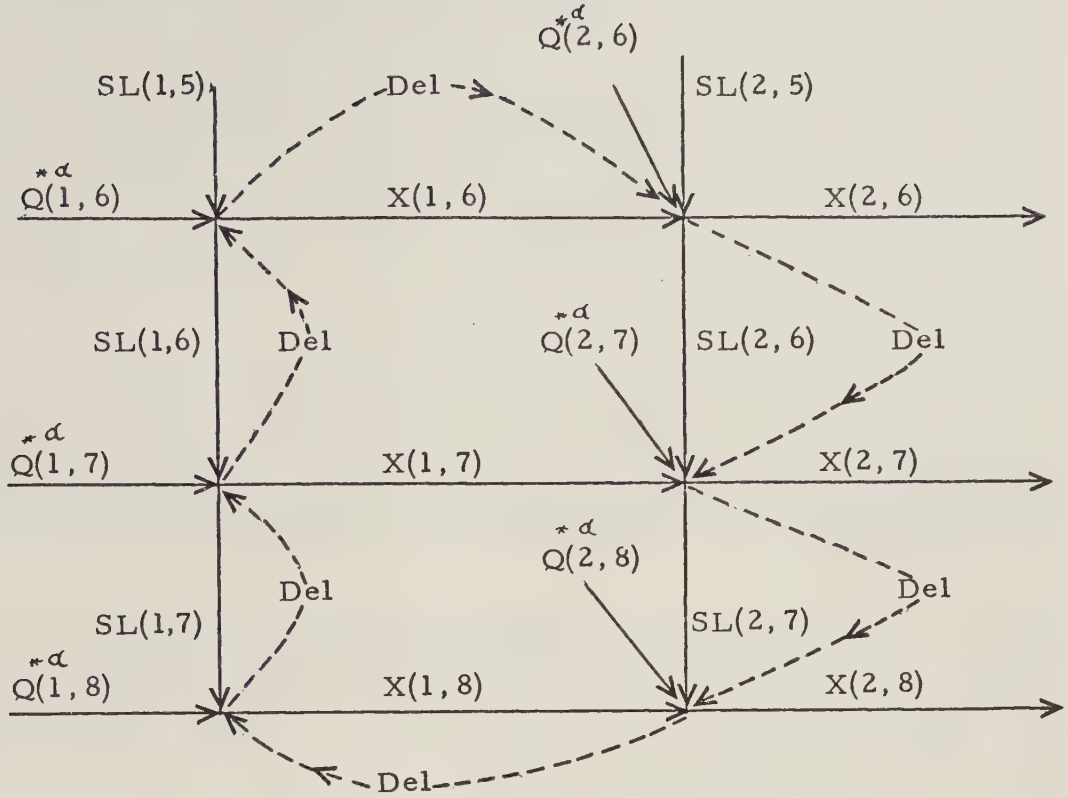

Fig 5a Part of the network corresponding to low cumulative inflows

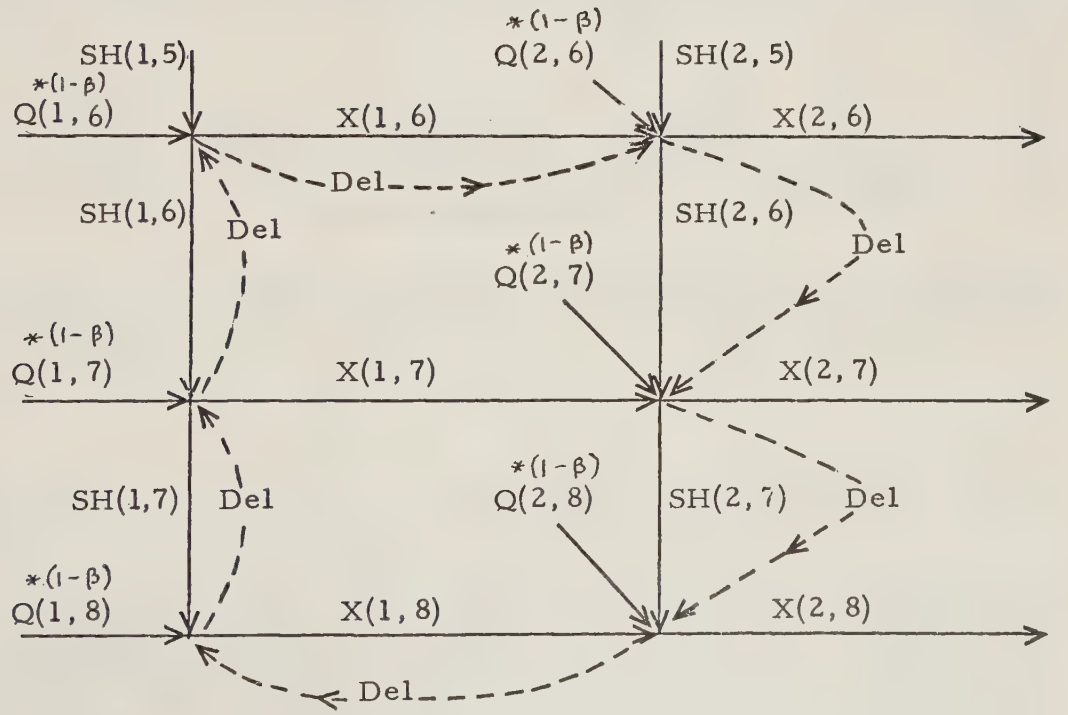

Fig 5b Part of the network corresponding to high cumulative inflows

We assumed perfect knowledge of the sums of inflows during spring (weeks 13-24 for Landösjön, weeks 15-26 otherwise), and used a stochastic model for Landösjön also. The stochastic models used at this stage of our research were of a similar kind to the new models but slightly different. For example only two non-spring seasons were used instead of three in the present versions, both the new and the old versions are described in Willén (1979). $\alpha = 0.05$ and $\beta = 0.10$ were used, 250 one-year series were generated with first time step corresponding to week 18 year 1, and $Q^{**0.05}$ and $Q^{**0.90}$ were computed. The optimization model was run with Smin (I, NT) = S(I, 0) = 7.5% of Smax (I, NT) and with Smin (I, NT) = S(I, 0) = 10% of Smax (I, NT).

In both cases we obtained PN1 > PN2 and feasible and optimal solutions were found. If S(I, 0) = 50% of Smax (I, NT) was used, no feasible solution existed because PN2 > PN1. In that case we had to use a higher PN in the earlier parts of the planning period and thus a lower PN during the rest of the time span.

Later the new versions of our stochastic models were applied, and the number of generated sequences were increased to 1000, see section 5.3. But because of the fair quality of inflow data for Landösjön, just the historical series was used for this reservoir. And $\alpha = 0.05$ is rather restrictive, so α was changed to $\alpha = 0.10$.

We again started with k=1 at week 18 year 1 and tried one optimization for a whole year, i.e. NT=52 and last step corresponded to week 17 year 2.

S(I, 0) were somewhat arbitrarily put equal to 30.0, 40,0, 30,0, 30,0, 40.0 VE for I=1,2,3,4,5 respectively and Smin (I, NT) were put equal to S(I, 0).

The initial states of the stochastic processes of inflows correspond to the years 1949 (wet), 1956 (normal), 1970 (dry) and 1973 (wet) respectively, and were determined according to the method described in section 5.3. I.e. we tried four one-year optimization problems.

All these problems were infeasible however. We obtained

$$PN2 > PN1 \tag{122}$$

and we also had

$$QS^{*(1-\beta)} - QS^{*\alpha} > Smax-Smin \tag{123}$$

for Burvattnet and Korsvattnet for k=52, and for Landösjön for several time steps, the earliest one was k=6 which corresponds to week 23 year 1. The reason for obtaining (122) and (123) is that when the assumption of pre-known sum of remaining spring inflows is abolished, the variability of QS increases significantly, which means that larger active storage capacities would be required in order to attain feasibility with the SRP-approach.

It should be emphasized that the infeasibility obtained just concerns our problem formulation; of course the operation problem of the real world is feasible.

If we want to achieve feasible problems, we have to change the problem formulation, one possibility is to increase α or β. As flood protection generally is no great problem here, it is reasonable to change β instead of α. In case of high inflows, changing of predetermined release usually does not cause any problems other than difficulties in passing all the water through the turbines. In addition, as high inflows probably are followed by rather high inflows (owing to the persistence), firm power generation will seldom cause problems in this case, instead we will probably be able to sell surplus power.

However, during the snow-melt season one has to be careful about risks of overflow, because during this period high inflows may mean that less snow-melt (and inflow) remains for later time steps.

That is we choose to increase β, but instead of determining how much to increase β, the increases are made by decreasing $QS^{*(1-\beta)}$. This is possible, since $QS^{*(1-\beta)}$ is a monotone decreasing function of β. As

$$QS^{*(1-\beta)} - QS^{*\alpha} \leq \text{Smax-Smin} \tag{124}$$

is a necessary condition for feasibility, one possibility is to decrease $QS^{*(1-\beta)}$ by

$$QKKI = (QS^{*(1-\beta)} - QS^{*\alpha}) - (\text{Smax-Smin}) \tag{125}$$

whenever eq (123) holds.

But this would lead to an optimization problem with fixed SH(I,k) and SL(I,k) for the values of I and k where eq (125) were applied. A certain degree of freedom for the optimization was desired however, so instead we compared DQ with DH where

$$DQ = QS^{*(1-\beta)} - QS^{*\alpha}$$

$$DH = 0.975 \cdot (\text{Smax-Smin})$$

We made these comparisons separately for each reservoir and started with the first time step. If for reservoir I we obtained DQ $>$ DH for this time step, then $QS^{*(1-\beta)}$ was decreased by

$$QKK = DQ - DH$$

for this and all subsequent time steps. We carried out the same procedure for k=2,...., and finally for k = NT.

The increases in β corresponding to these changes were not computed.

For Stora Mjölkvattnet and Övre Oldsjön no changes were necessary, for Burvattnet and Korsvattnet only QS for the last time step was changed, but for Landösjön many QS had to be modified, the earliest one was for k=6 which corresponds to week 23. This is in the latest stages of snowmelt for Landösjön in ordinary years, but the corrections for the snow-melt season

were rather small.

In spite of these modifications we still obtained PN2 > PN1 however. To deal with this the idea of having just one PN for the whole planning period was abolished. Instead PN = PN2 was used for k=1,2,....,KHR, where KHR is the time step corresponding to PN2 in eq (119), i.e. KHR is the critical time step considering overflow and spillage. At this critical time step, KHR, we must have

$$\sum_{k=1}^{KHR} X(I,k) = S(I,0) + QS^{*(1-\beta)}(I,KHR) - Smax(I,KHR) \qquad I=1,3 \tag{126}$$

$$\sum_{k=1}^{KHR} X(I,k) = \sum_{ID=I}^{I+1} (S(ID,0) + QS^{*(1-\beta)}(ID,KHR) - Smax(ID,KHR)) \qquad I = 2,4 \tag{127}$$

$$\sum_{k=1}^{KHR} X(5,k) = \sum_{I=1}^{5} (S(I,0) + QS^{*(1-\beta)}(I,KHR) - Smax(I,KHR)) \tag{128}$$

because in eq (118) we must have equality, and then all inequalities from which this equation is derived must have equality.

We then analyze the remaining time period, which is the time steps k = KHR+1,....,NT. For these time steps we should subtract the sums in eqs (126-128) from the resources of the plants (see discussion following eq (112)). Thus for reservoir no 1 we should have:

$$\sum_{k=KHR+1}^{T} X(1,k) \leq S(1,0) + QS^{*\alpha}(1,T) - Smin(1,T) - (S(1,0) + QS^{*(1-\beta)}(1,KHR) - Smax(1,KHR)) \tag{129}$$

and for reservoir 2 we should have

$$\sum_{k=KHR+1}^{T} X(2,k) \leq \sum_{I=1}^{2} (S(I,0) + QS^{*\alpha}(I,T) - Smin(I,T)) - \sum_{I=1}^{2} (S(I,0) - QS^{*(1-\beta)}(I,KHR) - Smax(I,KHR)) \tag{130}$$

where T=KHR+1, KHR+2,....,NT.

The upper limits for the sums of X(3,k), X(4,k) and X(5,k) are modified in the same way, this is also done for the lower limits of the sums of releases for each reservoir for this remaining time period.

The new PN1 and PN2 are computed for this time period by appropriate modifications of eqs (114) and (119). If we now obtain PN2 $\leq$ PN1, we try to follow

$$PN = \text{(the new value of) PN1} \tag{131}$$

for these time steps. This new value of PN1 is lower than that followed for the time steps up to k=KHR.

If we still have PN1 $<$ PN2, we repeat the procedure and thus divide the remaining part of the planning period into two parts, one with PN = (the new value of PN2) and the rest with a lower value of PN.

The problem were still infeasible however. The requirements of $Smin(I,NT) = S(I,0)$ together with $X(I,k) \geq Xmin(I,k)$ prevented us from releasing as much water as would be required to have SH $\leq$ Smax for some time steps preceding NT. (For all time steps except NT we used Smin=0.)

This problem appeared at Burvattnet in all four cases and for a period starting about new year and ending with time step NT-1, and for Korsvattnet a few time steps for the cases corresponding to start in years 1956 and 1970.

As the constraints against shortage are considered more important than those against spillage and overflow (see discussion above), we accepted to have SH $>$ Smax in these cases instead of violating other restrictions.

The results for the case corresponding to start at week 18 year 1949 is shown in Table 4. The values of PN obtained and followed are:

$1.85 \cdot 10^4$ Mwh	weeks 18-31
$1.14 \cdot 10^4$ Mwh	weeks 32-34
$0.93 \cdot 10^4$ Mwh	weeks 35-44
$0.74 \cdot 10^4$ Mwh	weeks 45 - week 17 year 2.

Here an efficiency of 100% is assumed for simplicity when presenting the results.

For year 1970 another kind of infeasibility arose, in spite of all the modifications made. For Landösjön the condition SH $\leq$ Smax was already violated for k=4, which corresponds to week 21, i.e. in the spring period. This was obtained even if we put $X(2,k) = Xmin(2,k)$ and $X(4,k) = Xmin(4,k)$ for k=1,2,3,4 and allocated all the remaining power production (according to the PN derived above) to the fictitious plant below Landösjön.

To deal with this we simply increased the power production for those weeks it was needed and compensated by decreasing it the next week. These corrections only influenced weeks 21-26.

An alternative would be to accept having SH $>$ Smax for these time steps - compare discussion above. But as this occurs in the snow-melt season, high inflows may very well mean just an unusual time distribution of

spring inflow, so high inflows are not necessarily followed by rather high ones in this case. The infeasibility which derives from the impossibility to use one PN for all time steps could also be handled by changing the production constraints (eqs (61) and (62)) into

$$HL \cdot X(2,k)+HO \cdot X(4,k)+HLA \cdot X(5,k) \geq B_k \cdot PN \qquad (132)$$

and applying e.g. the Simplex-method (see for example Lasdon (1970) for a description of this algorithm). But the other kinds of infeasibility cannot be dealt with in this way.

Moreover, this problem formulation gives no answer to the question of how to distribute the excess power (excess relative to $B_k \cdot PN$) over the weeks.

These four problems (corresponding to years 1949, 1956, 1970 and 1973) were combined with simulation studies. Because we could not obtain feasible solutions with our SRP-policy and because in all real cases we must be ready to change our predetermined scheme of operation, we applied recalculation of the "optimum" policy with different starting times but with week 17 year 2 as the last time step. First simulation was applied so that the initial state of reservoirs (storages) was used and the real inflow that happened these four years were used as input. Also for these simulations the time period lasted until week 17 year 2.

The predetermined releases were modified whenever storage below SL or above S´max was obtained, where

$$S'max = \text{Min } (SH, Smax) \qquad (133)$$

For the upper reservoirs (Burvattnet and Korsvattnet) from which the releases do not enter any power-plants before reaching Stora Mjölkvattnet and Övre Oldsjön respectively, we just increased or decreased the release when needed to keep storage within the desired limits. These corrections were sufficient for these two reservoirs. After having dealt with these two reservoirs, the nearest lower reservoirs, Stora Mjölkvattnet and Övre Oldsjön were treated. In case of storage outside the desired limits, X was modified for the reservoir and compensation with the other reservoir was tried in order not to change the power production. But if (complete) compensation was impossible, the power production was changed so that the storage remained within desired limits. Finally Landösjön was treated.

In case of $S(5,k) > S'max(5,k)$, we just increased $X(5,k)$ (and the power production) without trying to compensate in the other plants.

But if $S(5,k) < SL(5,k)$ was obtained, $X(5,k)$ was decreased and the power deficit obtained was covered by the other plants. These corrections were made so that we then had $S(5,k) = SL(5,k)$. In all cases we succeeded in fully compensating power deficits, but sometimes the power production was increased compared to that determined in the optimization program. This is not surprising because when making the optimization problem feasible, we retained all constraints against shortage but deleted some constraints against overflow and spillage. These corrections were made step by step

in the simulation program, starting with time step one.

This simulation program was combined with the optimization model in the following way for the four years 1949, 1956, 1970 and 1973.

1) A one-year optimization with start at week 18 and ending at week 17 next year were made as described above.
2) Simulation with the real inflows that occurred using the same initial storages as in step 1 and using the release scheme derived in that step as the initial release scheme.
3) Rerunning the optimization model with the same ending time step (week 17 next year) but starting with k=1 at week 27 year 1. The values of S(I,26), (I=1,2,3,4,5), obtained in step 2) were used as initial storages.
4) A new simulation covering the same time period and with the same initial storages as in step 3).
5) Rerunning the optimization model still with the same ending time step, but starting at week 36 year 1. The values of S(I,35) from step 4) were used as initial storages.
6) Simulation covering the same time period as step 5) and in the same way as in step 4).
7) Rerunning the optimization model with start at week 1 year 2 etc. (This step was not performed for year 1973-74, due to lack of inflow data for year 1974.)
8) Simulation covering the same time period etc. (Not for year 1973-74.)

In the re-computations of the optimal policy starting at week 27, we had to apply eq (126) for Landösjön for several time steps, the earliest modification was for k=17 (week 43 year 1) in all four cases. For Burvattnet the last time step needed correction of this type except when starting in year 1970. Stora Mjölkvattnet, Korsvattnet and Övre Oldsjön needed no corrections.

In the re-computations starting at weeks 36 year 1 or week 1 year 2 no modifications of QS were needed.

In figures 6 a-d are shown the values of PN (n o r m a l electricity production, see eq (61)) in Mwh obtained from the optimization programs. The numbers in parenthesis denote the starting time of the optimization period. Also shown are the increases in power production made in the simulations, these increases are denoted by vertical arrows. Due to the re-calculations of the operation model made at several weeks of the year, we only show the increases relative to the latest recomputed optimization program.

The storages in the different reservoirs which would result from our combined optimization-simulation approach are shown in figures 7 a-h.

Table 4a. Result of a one-year optimization corresponding to start at week 18 year 1949. Unit: VE

Here QL means $Q^{**0.10}$ and QH means $Q^{**0.90}$.

BURVATTNET

INITIAL STORAGE 30.0 VE

WEEK	STEP	XMIN	X	XMAX	QL	SMIN	SL	QH	SH	SMAX
18	1	0.7	8.2	20.0	3.5	0.0	25.3	23.8	45.6	99.3
19	2	0.7	9.4	20.0	7.3	0.0	23.2	32.7	68.9	99.3
20	3	0.7	9.3	20.0	10.0	0.0	24.0	27.8	87.4	99.3
21	4	0.7	9.2	20.0	13.8	0.0	28.6	18.6	96.8	99.3
22	5	1.3	11.8	20.0	18.6	0.0	35.4	14.3	99.3	99.3
23	6	2.0	11.4	20.0	21.5	0.0	45.4	11.4	99.3	99.3
24	7	2.0	11.6	20.0	16.4	0.0	50.2	11.6	99.3	99.3
25	8	2.0	10.0	20.0	15.0	0.0	55.2	10.0	99.3	99.3
26	9	2.0	10.4	20.0	10.3	0.0	55.1	10.4	99.3	99.3
27	10	2.0	7.5	20.0	5.3	0.0	52.8	7.5	99.3	99.3
28	11	2.0	6.4	20.0	3.5	0.0	50.0	6.4	99.3	99.3
29	12	2.0	5.9	20.0	2.8	0.0	46.9	5.9	99.3	99.3
30	13	2.0	5.5	20.0	2.8	0.0	44.3	5.5	99.3	99.3
31	14	2.0	5.3	20.0	2.3	0.0	41.3	5.3	99.3	99.3
32	15	2.0	4.2	20.0	2.2	0.0	39.2	4.2	99.3	99.3
33	16	2.0	4.4	20.0	2.5	0.0	37.4	3.9	98.8	99.3
34	17	2.0	4.6	20.0	2.0	0.0	34.9	5.0	99.3	99.3
35	18	2.0	4.6	20.0	2.4	0.0	32.6	4.3	99.0	99.3
36	19	2.0	4.8	20.0	3.5	0.0	31.3	3.7	97.9	99.3
37	20	2.0	4.8	20.0	3.8	0.0	30.3	6.1	99.2	99.3
38	21	2.0	5.0	20.0	2.3	0.0	27.6	5.1	99.3	99.3
39	22	2.0	4.9	20.0	3.7	0.0	26.3	4.0	98.4	99.3
40	23	0.7	5.0	20.0	3.5	0.0	24.8	4.5	98.0	99.3
41	24	0.7	5.0	20.0	3.8	0.0	23.7	6.0	98.9	99.3
42	25	0.7	5.1	20.0	3.7	0.0	22.4	4.1	97.9	99.3
43	26	0.7	5.7	20.0	4.5	0.0	21.2	7.0	99.3	99.3
44	27	0.7	3.8	20.0	2.9	0.0	20.2	3.8	99.3	99.3
45	28	0.7	3.0	20.0	2.3	0.0	19.5	3.0	99.3	99.3
46	29	0.7	2.8	20.0	2.3	0.0	19.0	2.8	99.3	99.3
47	30	0.7	2.3	20.0	1.6	0.0	18.3	2.3	99.3	99.3
48	31	0.7	2.1	20.0	1.2	0.0	17.4	2.1	99.3	99.3
49	32	0.7	2.6	20.0	1.2	0.0	16.0	2.6	99.3	99.3
50	33	0.7	0.9	20.0	2.0	0.0	17.1	0.9	99.3	99.3
51	34	0.7	1.9	20.0	1.0	0.0	16.2	1.9	99.3	99.3
52	35	0.7	1.5	20.0	1.9	0.0	16.6	1.5	99.3	99.3
1	36	0.7	0.9	20.0	1.6	0.0	17.3	1.1	99.5	99.3
2	37	0.7	0.7	20.0	1.2	0.0	17.8	1.3	100.1	99.3
3	38	0.7	0.7	20.0	1.4	0.0	18.5	1.8	101.2	99.3
4	39	0.7	0.7	20.0	1.1	0.0	18.9	1.9	102.4	99.3
5	40	0.7	0.7	20.0	1.5	0.0	19.7	1.1	102.8	99.3
6	41	0.7	0.7	20.0	1.8	0.0	20.7	1.2	103.3	99.3
7	42	0.7	0.7	20.0	1.3	0.0	21.4	1.1	103.7	99.3
8	43	0.7	0.7	20.0	1.4	0.0	22.0	2.3	105.3	99.3
9	44	0.7	0.7	20.0	1.3	0.0	22.7	0.8	105.4	99.3
10	45	0.7	0.7	20.0	2.0	0.0	24.0	1.8	106.5	99.3
11	46	0.7	0.7	20.0	1.1	0.0	24.3	2.3	108.0	99.3
12	47	0.7	0.7	20.0	0.8	0.0	24.5	2.2	109.5	99.3
13	48	0.7	0.7	20.0	2.0	0.0	25.8	1.2	110.0	99.3
14	49	0.7	0.7	20.0	0.9	0.0	26.0	1.6	111.0	99.3
15	50	0.7	0.7	20.0	1.2	0.0	26.5	1.7	112.0	99.3
16	51	0.7	0.7	20.0	2.4	0.0	28.3	1.6	112.9	99.3
17	52	0.7	0.7	20.0	2.4	30.0	30.0	-14.6	97.6	99.3

Table 4b. Result of a one-year optimization corresponding to start at week 18 year 1949.
Unit: VE

Here QL means $Q^{**0.10}$ and QH means $Q^{**0.90}$.

MJOELKVATTNET

INITIAL STORAGE 40.0 VE

EEK	STEP	XMIN	X	XMAX	QL	SMIN	SL	QH	SH	SMAX
18	1	1.2	10.1	34.0	4.7	0.0	42.8	31.8	70.0	268.3
19	2	1.2	11.5	34.0	9.8	0.0	50.5	43.8	111.6	268.3
20	3	1.2	11.4	34.0	13.4	0.0	61.8	37.2	146.7	268.3
21	4	1.2	11.2	34.0	18.5	0.0	78.2	24.8	169.5	268.3
22	5	1.2	11.2	34.0	24.9	0.0	103.7	19.1	189.2	268.3
23	6	1.2	11.2	34.0	28.7	0.0	132.6	15.3	204.7	268.3
24	7	1.2	9.9	34.0	21.9	0.0	156.2	15.5	221.9	268.3
25	8	1.2	1.2	34.0	20.1	0.0	185.1	13.4	244.1	268.3
26	9	1.2	4.6	34.0	13.8	0.0	204.8	14.0	264.0	268.3
27	10	1.2	16.4	34.0	7.0	0.0	202.9	10.0	265.1	268.3
28	11	1.2	13.1	34.0	4.7	0.0	200.9	8.5	266.9	268.3
29	12	1.2	15.5	34.0	3.7	0.0	195.0	7.8	265.1	268.3
30	13	1.2	11.1	34.0	3.8	0.0	193.2	7.3	266.8	268.3
31	14	1.2	10.8	34.0	3.1	0.0	190.7	7.1	268.3	268.3
32	15	1.2	9.9	34.0	2.9	0.0	187.9	5.7	268.3	268.3
33	16	1.2	10.2	34.0	3.4	0.0	185.4	5.2	267.7	268.3
34	17	1.2	10.7	34.0	2.7	0.0	182.1	6.7	268.3	268.3
35	18	1.2	10.6	34.0	3.2	0.0	179.2	5.8	268.0	268.3
36	19	1.2	11.3	34.0	4.7	0.0	177.4	5.0	266.5	268.3
37	20	1.2	11.3	34.0	5.0	0.0	176.0	8.1	268.2	268.3
38	21	1.2	11.6	34.0	3.0	0.0	172.4	6.8	268.3	268.3
39	22	1.2	11.5	34.0	4.9	0.0	170.7	5.4	267.1	268.3
40	23	1.2	11.7	34.0	4.7	0.0	168.7	6.1	266.5	268.3
41	24	1.2	11.8	34.0	5.3	0.0	167.2	8.0	267.8	268.3
42	25	1.2	11.9	34.0	5.0	0.0	165.4	5.4	266.4	268.3
43	26	1.2	13.2	34.0	6.0	0.0	163.8	9.4	268.3	268.3
44	27	1.2	9.0	34.0	3.8	0.0	162.5	5.1	268.3	268.3
45	28	1.2	7.6	34.0	3.1	0.0	161.0	4.0	267.7	268.3
46	29	1.2	9.3	34.0	3.1	0.0	157.6	3.8	265.0	268.3
47	30	1.2	9.5	34.0	2.1	0.0	152.5	3.1	260.9	268.3
48	31	1.2	9.5	34.0	1.6	0.0	146.7	2.8	256.2	268.3
49	32	1.2	9.6	34.0	1.6	0.0	141.2	3.4	252.6	268.3
50	33	1.2	9.7	34.0	2.7	0.0	135.1	1.3	245.1	268.3
51	34	1.2	5.1	34.0	1.4	0.0	133.3	2.6	244.5	268.3
52	35	1.2	5.1	34.0	2.5	0.0	132.2	2.0	242.9	268.3
1	36	1.2	5.1	34.0	2.2	0.0	130.2	1.4	240.2	268.3
2	37	1.2	5.1	34.0	1.6	0.0	127.3	1.8	237.5	268.3
3	38	1.2	9.6	34.0	1.9	0.0	120.3	2.4	231.0	268.3
4	39	1.2	9.5	34.0	1.5	0.0	113.1	2.6	224.8	268.3
5	40	1.2	9.4	34.0	2.0	0.0	106.4	1.4	217.6	268.3
6	41	1.2	9.3	34.0	2.4	0.0	100.1	1.6	210.6	268.3
7	42	1.2	9.1	34.0	1.8	0.0	93.5	1.5	203.6	268.3
8	43	1.2	9.1	34.0	1.9	0.0	86.9	3.0	198.3	268.3
9	44	1.2	9.0	34.0	1.8	0.0	80.4	1.1	191.1	268.3
10	45	1.2	8.9	34.0	2.7	0.0	74.9	2.4	185.3	268.3
11	46	1.2	8.9	34.0	1.4	0.0	68.2	3.0	180.2	268.3
12	47	1.2	8.8	34.0	1.1	0.0	61.2	2.9	175.0	268.3
13	48	1.2	7.5	34.0	2.7	0.0	57.1	1.6	169.8	268.3
14	49	1.2	7.4	34.0	1.3	0.0	51.7	2.2	165.3	268.3
15	50	1.2	7.3	34.0	1.6	0.0	46.7	2.3	161.0	268.3
16	51	1.2	7.3	34.0	3.3	0.0	43.3	2.1	156.6	268.3
17	52	1.2	7.3	34.0	3.3	40.0	40.0	4.7	154.6	268.3

Table 4c. **Result of a one-year optimization corresponding to start at week 18 year 1949.**
Unit: VE

Here QL means $Q^{**0.10}$ and QH means $Q^{**0.90}$.

KORSVATTNET

INITIAL STORAGE 30.0 VE

WEEK	STEP	XMIN	X	XMAX	QL	SMIN	SL	QH	SH	SMAX
18	1	0.6	6.6	20.0	1.7	0.0	25.1	14.9	38.3	90.0
19	2	0.6	7.5	20.0	4.1	0.0	21.6	21.9	52.7	90.0
20	3	0.6	7.4	20.0	8.6	0.0	22.8	16.2	61.5	90.0
21	4	0.6	7.4	20.0	11.0	0.0	26.4	16.0	70.2	90.0
22	5	1.0	7.4	20.0	16.3	0.0	35.3	17.1	79.9	90.0
23	6	1.5	7.4	20.0	19.2	0.0	47.1	15.1	87.6	90.0
24	7	1.5	10.1	20.0	15.9	0.0	53.0	12.5	90.0	90.0
25	8	1.5	11.1	20.0	14.7	0.0	56.6	11.1	90.0	90.0
26	9	1.5	9.4	20.0	10.6	0.0	57.8	9.4	90.0	90.0
27	10	1.5	6.4	20.0	5.4	0.0	57.1	6.4	90.0	90.0
28	11	1.5	5.0	20.0	3.1	0.0	55.2	5.0	90.0	90.0
29	12	1.5	4.2	20.0	2.6	0.0	53.6	4.2	90.0	90.0
30	13	1.5	3.5	20.0	2.6	0.0	52.7	3.5	90.0	90.0
31	14	1.5	4.6	20.0	3.2	0.0	51.3	4.6	90.0	90.0
32	15	1.5	4.1	20.0	1.7	0.0	48.9	4.1	90.0	90.0
33	16	1.5	3.6	20.0	2.3	0.0	47.6	3.5	89.9	90.0
34	17	1.5	4.0	20.0	2.4	0.0	45.9	4.1	90.0	90.0
35	18	1.5	3.6	20.0	2.6	0.0	44.9	3.6	90.0	90.0
36	19	1.5	3.6	20.0	2.3	0.0	43.6	3.1	89.5	90.0
37	20	1.5	3.7	20.0	2.2	0.0	42.0	3.0	88.9	90.0
38	21	1.5	3.8	20.0	2.3	0.0	40.6	3.5	88.6	90.0
39	22	1.5	3.8	20.0	3.2	0.0	40.0	4.2	89.0	90.0
40	23	0.6	3.8	20.0	2.4	0.0	38.6	4.2	89.3	90.0
41	24	0.6	3.9	20.0	2.4	0.0	37.1	3.5	89.0	90.0
42	25	0.6	3.9	20.0	2.7	0.0	35.9	4.1	89.2	90.0
43	26	0.6	4.0	20.0	3.2	0.0	35.2	3.8	89.0	90.0
44	27	0.6	3.3	20.0	2.3	0.0	34.2	4.4	90.0	90.0
45	28	0.6	1.8	20.0	1.8	0.0	34.2	1.8	90.0	90.0
46	29	0.6	1.2	20.0	1.3	0.0	34.3	1.2	90.0	90.0
47	30	0.6	1.2	20.0	1.0	0.0	34.2	1.2	90.0	90.0
48	31	0.6	0.9	20.0	1.3	0.0	34.5	0.8	89.8	90.0
49	32	0.6	1.4	20.0	0.5	0.0	33.6	1.3	89.8	90.0
50	33	0.6	1.4	20.0	0.5	0.0	32.7	1.3	89.7	90.0
51	34	0.6	1.1	20.0	1.3	0.0	32.9	1.4	90.0	90.0
52	35	0.6	1.1	20.0	0.7	0.0	32.5	1.1	90.0	90.0
1	36	0.6	0.8	20.0	0.8	0.0	32.5	0.8	90.0	90.0
2	37	0.6	1.3	20.0	0.5	0.0	31.7	1.3	90.0	90.0
3	38	0.6	1.3	20.0	1.3	0.0	31.8	1.3	90.0	90.0
4	39	0.6	0.8	20.0	0.6	0.0	31.5	0.8	90.0	90.0
5	40	0.6	1.3	20.0	0.9	0.0	31.0	1.3	90.0	90.0
6	41	0.6	1.4	20.0	1.1	0.0	30.7	1.4	90.0	90.0
7	42	0.6	0.7	20.0	0.5	0.0	30.5	0.7	90.0	90.0
8	43	0.6	1.3	20.0	1.0	0.0	30.2	1.3	90.0	90.0
9	44	0.6	1.1	20.0	1.3	0.0	30.4	1.1	90.0	90.0
10	45	0.6	1.2	20.0	0.6	0.0	29.9	0.2	89.0	90.0
11	46	0.6	1.3	20.0	1.2	0.0	29.8	0.6	88.3	90.0
12	47	0.6	1.2	20.0	1.0	0.0	29.6	1.4	88.5	90.0
13	48	0.6	1.1	20.0	0.9	0.0	29.5	0.9	88.4	90.0
14	49	0.6	1.0	20.0	1.1	0.0	29.5	0.7	88.1	90.0
15	50	0.6	1.0	20.0	1.4	0.0	29.9	0.9	88.0	90.0
16	51	0.6	1.0	20.0	0.9	0.0	29.7	1.1	88.1	90.0
17	52	0.6	1.0	20.0	1.4	30.0	30.0	1.4	88.5	90.0

Table 4d. Result of a one-year optimization corresponding to start at week 18 year 1949.
Unit: VE
Here QL means $Q^{**0.10}$ and QH means $Q^{**0.90}$.

OEVRE OLDSJOEN

INITIAL STORAGE 30.0 VE

WEEK	STEP	XMIN	X	XMAX	QL	SMIN	SL	QH	SH	SMAX
18	1	0.0	0.5	31.0	2.1	0.0	38.2	18.8	54.8	204.6
19	2	0.0	0.0	31.0	5.1	0.0	50.9	27.5	89.9	204.6
20	3	0.0	0.0	31.0	10.8	0.0	69.1	20.4	117.7	204.6
21	4	0.0	0.0	31.0	13.8	0.0	90.3	20.2	145.3	204.6
22	5	0.0	0.0	31.0	20.4	0.0	118.1	21.5	174.1	204.6
23	6	0.0	10.0	31.0	24.1	0.0	139.5	18.9	190.3	204.6
24	7	0.0	15.9	31.0	20.0	0.0	153.7	15.7	200.3	204.6
25	8	0.0	20.6	31.0	18.5	0.0	162.6	13.9	204.6	204.6
26	9	0.0	21.2	31.0	13.3	0.0	164.2	11.8	204.6	204.6
27	10	0.0	14.4	31.0	7.1	0.0	163.2	8.0	204.6	204.6
28	11	0.0	11.2	31.0	3.9	0.0	160.9	6.2	204.6	204.6
29	12	0.0	9.5	31.0	3.3	0.0	158.9	5.3	204.6	204.6
30	13	0.0	9.3	31.0	3.3	0.0	156.3	4.3	203.1	204.6
31	14	0.0	8.8	31.0	4.0	0.0	156.0	5.8	204.6	204.6
32	15	0.0	9.2	31.0	2.1	0.0	153.0	5.1	204.6	204.6
33	16	0.0	8.1	31.0	2.9	0.0	151.3	4.4	204.5	204.6
34	17	0.0	9.1	31.0	3.0	0.0	149.2	5.2	204.6	204.6
35	18	0.0	8.1	31.0	3.2	0.0	147.9	4.5	204.6	204.6
36	19	0.0	8.1	31.0	2.8	0.0	146.3	3.9	204.0	204.6
37	20	0.0	8.3	31.0	2.7	0.0	144.3	3.8	203.2	204.6
38	21	0.0	8.4	31.0	2.9	0.0	142.6	4.4	202.9	204.6
39	22	0.0	8.6	31.0	4.0	0.0	141.8	5.3	203.3	204.6
40	23	0.0	8.6	31.0	3.0	0.0	140.0	5.2	203.7	204.6
41	24	0.0	8.7	31.0	3.0	0.0	138.2	4.4	203.3	204.6
42	25	0.0	8.8	31.0	3.3	0.0	136.6	5.1	203.5	204.6
43	26	0.0	8.1	31.0	4.0	0.0	136.6	4.7	204.2	204.6
44	27	0.0	8.3	31.0	2.9	0.0	134.5	5.5	204.6	204.6
45	28	0.0	6.1	31.0	2.3	0.0	132.5	2.3	202.6	204.6
46	29	0.0	7.5	31.0	1.7	0.0	127.8	1.5	197.7	204.6
47	30	0.0	7.6	31.0	1.3	0.0	122.6	1.5	192.8	204.6
48	31	0.0	7.7	31.0	1.6	0.0	117.4	1.0	186.9	204.6
49	32	0.0	7.8	31.0	0.6	0.0	111.6	1.6	182.2	204.6
50	33	0.0	7.8	31.0	0.6	0.0	105.8	1.7	177.4	204.6
51	34	0.0	4.1	31.0	1.6	0.0	104.5	1.7	176.2	204.6
52	35	0.0	4.1	31.0	0.8	0.0	102.3	1.4	174.6	204.6
1	36	0.0	4.1	31.0	1.0	0.0	100.0	0.9	172.2	204.6
2	37	0.0	4.2	31.0	0.7	0.0	97.8	1.6	171.0	204.6
3	38	0.0	8.5	31.0	1.7	0.0	92.2	1.6	165.4	204.6
4	39	0.0	9.2	31.0	0.7	0.0	84.6	1.0	158.1	204.6
5	40	0.0	9.0	31.0	1.1	0.0	78.0	1.7	152.1	204.6
6	41	0.0	8.8	31.0	1.3	0.0	72.0	1.8	146.5	204.6
7	42	0.0	7.8	31.0	0.6	0.0	65.4	0.8	140.2	204.6
8	43	0.0	8.5	31.0	1.3	0.0	59.5	1.6	134.6	204.6
9	44	0.0	8.4	31.0	1.6	0.0	53.8	1.4	128.6	204.6
10	45	0.0	8.2	31.0	0.8	0.0	47.6	0.2	121.8	204.6
11	46	0.0	8.1	31.0	1.6	0.0	42.2	0.7	115.7	204.6
12	47	0.0	8.2	31.0	1.3	0.0	36.5	1.8	110.5	204.6
13	48	0.0	6.2	31.0	1.2	0.0	32.6	1.2	106.6	204.6
14	49	0.0	1.7	31.0	1.4	0.0	33.3	0.9	106.8	204.6
15	50	0.0	6.0	31.0	1.7	0.0	30.0	1.2	103.0	204.6
16	51	0.0	0.0	31.0	1.1	0.0	32.1	1.4	105.5	204.6
17	52	0.0	4.9	31.0	1.7	30.0	30.0	2.1	103.8	204.6

Table 4e. Result of a one-year optimization corresponding to start at week 18 year 1949.
Unit: VE
Here QL means $Q^{**0.10}$ and QH means $Q^{**0.90}$

LANDOESJOEN

INITIAL STORAGE 40.0 VE

WEEK	STEP	XMIN	X	XMAX	CL	SMIN	SL	QH	SH	SMAX
18	1	4.4	51.0	200.0	13.5	0.0	13.2	95.5	95.1	269.7
19	2	4.4	58.6	200.0	52.4	0.0	18.4	94.3	142.2	269.7
20	3	4.4	58.0	200.0	67.0	0.0	38.8	111.8	207.4	269.7
21	4	4.4	57.3	200.0	25.9	0.0	18.6	96.6	257.8	269.7
22	5	8.7	57.3	200.0	36.1	0.0	8.6	58.0	269.7	269.7
23	6	15.0	47.3	200.0	24.2	0.0	6.7	26.0	269.7	269.7
24	7	15.0	51.4	200.0	22.4	0.0	3.5	20.6	264.7	269.7
25	8	15.0	39.2	200.0	27.0	0.0	13.1	22.4	269.7	269.7
26	9	15.0	34.6	200.0	39.9	0.0	44.1	8.9	269.7	269.7
27	10	15.0	37.5	200.0	15.1	0.0	52.5	6.7	269.7	269.7
28	11	15.0	31.4	200.0	2.8	0.0	48.3	7.1	269.7	269.7
29	12	15.0	31.2	200.0	18.3	0.0	60.4	6.1	269.7	269.7
30	13	15.0	39.2	200.0	12.0	0.0	53.6	6.0	256.8	269.7
31	14	15.0	46.6	200.0	2.3	0.0	29.0	39.8	269.7	269.7
32	15	15.0	26.7	200.0	2.2	0.0	23.7	4.6	266.7	269.7
33	16	15.0	29.8	200.0	2.4	0.0	14.6	9.4	264.7	269.7
34	17	15.0	29.8	200.0	2.2	0.0	6.7	15.0	269.7	269.7
35	18	15.0	22.2	200.0	2.3	0.0	5.7	2.3	268.6	269.7
36	19	15.0	22.8	200.0	2.8	0.0	5.1	2.8	268.1	269.7
37	20	15.0	23.0	200.0	3.0	0.0	4.6	3.0	267.6	269.7
38	21	15.0	23.5	200.0	2.3	0.0	3.4	2.3	266.3	269.7
39	22	15.0	23.7	200.0	2.9	0.0	2.7	2.9	265.7	269.7
40	23	4.4	23.9	200.0	2.8	0.0	1.9	2.8	264.9	269.7
41	24	4.4	24.1	200.0	3.0	0.0	1.4	3.0	264.3	269.7
42	25	4.4	24.3	200.0	2.9	0.0	0.6	2.9	263.6	269.7
43	26	4.4	24.7	200.0	7.5	0.0	4.7	7.5	267.6	269.7
44	27	4.4	20.7	200.0	5.4	0.0	6.7	5.4	269.7	269.7
45	28	4.4	17.6	200.0	2.3	0.0	5.1	2.3	268.1	269.7
46	29	4.4	21.6	200.0	2.3	0.0	2.7	2.3	265.6	269.7
47	30	4.4	21.9	200.0	3.8	0.0	1.7	3.8	264.6	269.7
48	31	4.4	22.1	200.0	11.7	0.0	8.5	8.3	268.0	269.7
49	32	4.4	22.3	200.0	5.2	0.0	8.8	4.7	267.8	269.7
50	33	4.4	22.4	200.0	2.1	0.0	6.0	4.3	267.2	269.7
51	34	4.4	11.8	200.0	1.7	0.0	5.1	2.2	266.8	269.7
52	35	4.4	11.8	200.0	1.4	0.0	3.8	2.6	266.8	269.7
1	36	4.4	11.8	200.0	1.2	0.0	2.4	1.2	265.4	269.7
2	37	4.4	11.9	200.0	1.7	0.0	1.5	1.7	264.5	269.7
3	38	4.4	21.5	200.0	1.9	0.0	0.0	1.9	263.0	269.7
4	39	4.4	20.4	200.0	1.7	0.0	0.0	1.7	263.0	269.7
5	40	4.4	20.3	200.0	2.0	0.0	0.0	1.8	262.8	269.7
6	41	4.4	20.2	200.0	2.1	0.0	0.0	2.1	262.7	269.7
7	42	4.4	20.8	200.0	3.8	0.0	0.0	1.9	260.8	269.7
8	43	4.4	19.9	200.0	2.3	0.0	0.0	2.8	261.2	269.7
9	44	4.4	19.6	200.0	2.2	0.0	0.0	1.8	260.8	269.7
10	45	4.4	19.7	200.0	2.6	0.0	0.0	2.1	260.3	269.7
11	46	4.4	19.5	200.0	2.5	0.0	0.0	5.2	263.0	269.7
12	47	4.4	19.2	200.0	2.2	0.0	0.0	2.2	263.0	269.7
13	48	4.4	17.3	200.0	3.6	0.0	0.0	3.6	263.0	269.7
14	49	4.4	21.3	200.0	12.2	0.0	0.0	7.4	258.1	269.7
15	50	4.4	16.8	200.0	3.5	0.0	0.0	6.2	260.8	269.7
16	51	4.4	22.8	200.0	16.8	0.0	1.3	6.7	252.0	269.7
17	52	4.4	17.9	200.0	44.4	40.0	40.0	17.7	264.0	269.7

Table 4f. Result of a one-year optimization corresponding to start at week 18 year 1949.

		PRODUCTION IN MWH			
STEP	WEEK	LAANGSAA	OLOAA	FICTIOUS	TOTAL
1	18	0.31769E+04	0.21936E+03	0.20933E+05	0.24329E+05
2	19	0.36137E+04	0.0	0.24061E+05	0.27674E+05
3	20	0.35740E+04	0.0	0.23796E+05	0.27370E+05
4	21	0.35343E+04	0.0	0.23532E+05	0.27066E+05
5	22	0.35343E+04	0.0	0.23532E+05	0.27066E+05
6	23	0.35343E+04	0.41181E+04	0.19414E+05	0.27066E+05
7	24	0.31279E+04	0.65076E+04	0.21088E+05	0.30715E+05
8	25	0.37774E+03	0.84583E+04	0.16101E+05	0.24937E+05
9	26	0.14406E+04	0.86840E+04	0.14204E+05	0.24329E+05
10	27	0.51634E+04	0.59067E+04	0.15388E+05	0.26458E+05
11	28	0.41188E+04	0.46012E+04	0.12872E+05	0.21592E+05
12	29	0.48792E+04	0.39161E+04	0.12797E+05	0.21592E+05
13	30	0.34878E+04	0.38278E+04	0.16101E+05	0.23417E+05
14	31	0.34151E+04	0.36249E+04	0.19114E+05	0.26154E+05
15	32	0.31244E+04	0.37680E+04	0.10942E+05	0.17835E+05
16	33	0.32253E+04	0.33261E+04	0.12222E+05	0.18773E+05
17	34	0.33538E+04	0.37471E+04	0.12235E+05	0.19336E+05
18	35	0.33511E+04	0.33421E+04	0.90965E+04	0.15790E+05
19	36	0.35705E+04	0.33175E+04	0.93615E+04	0.16250E+05
20	37	0.35435E+04	0.34094E+04	0.94499E+04	0.16403E+05
21	38	0.36446E+04	0.34384E+04	0.96265E+04	0.16709E+05
22	39	0.36081E+04	0.35399E+04	0.97148E+04	0.16863E+05
23	40	0.36760E+04	0.35369E+04	0.98031E+04	0.17016E+05
24	41	0.37091E+04	0.35688E+04	0.98914E+04	0.17169E+05
25	42	0.37422E+04	0.36006E+04	0.99798E+04	0.17323E+05
26	43	0.41656E+04	0.33072E+04	0.10156E+05	0.17629E+05
27	44	0.28221E+04	0.34158E+04	0.84786E+04	0.14717E+05
28	45	0.23948E+04	0.25200E+04	0.72366E+04	0.12151E+05
29	46	0.29270E+04	0.30797E+04	0.88450E+04	0.14852E+05
30	47	0.29753E+04	0.31307E+04	0.89912E+04	0.15097E+05
31	48	0.29995E+04	0.31561E+04	0.90643E+04	0.15220E+05
32	49	0.30237E+04	0.31816E+04	0.91374E+04	0.15343E+05
33	50	0.30479E+04	0.32070E+04	0.92105E+04	0.15465E+05
34	51	0.15965E+04	0.16799E+04	0.48245E+04	0.81009E+04
35	52	0.15965E+04	0.16799E+04	0.48245E+04	0.81009E+04
36	1	0.15965E+04	0.16799E+04	0.48245E+04	0.81009E+04
37	2	0.16207E+04	0.17053E+04	0.48976E+04	0.82237E+04
38	3	0.30237E+04	0.34948E+04	0.88241E+04	0.15343E+05
39	4	0.29753E+04	0.37708E+04	0.83511E+04	0.15097E+05
40	5	0.29512E+04	0.36795E+04	0.83438E+04	0.14974E+05
41	6	0.29270E+04	0.36181E+04	0.83067E+04	0.14852E+05
42	7	0.28786E+04	0.32049E+04	0.85228E+04	0.14606E+05
43	8	0.28544E+04	0.34720E+04	0.81570E+04	0.14483E+05
44	9	0.28302E+04	0.34669E+04	0.80636E+04	0.14361E+05
45	10	0.28060E+04	0.33448E+04	0.80871E+04	0.14238E+05
46	11	0.27818E+04	0.33414E+04	0.79919E+04	0.14115E+05
47	12	0.27576E+04	0.33697E+04	0.78651E+04	0.13992E+05
48	13	0.23706E+04	0.25394E+04	0.71186E+04	0.12029E+05
49	14	0.23222E+04	0.70466E+03	0.87562E+04	0.11783E+05
50	15	0.22980E+04	0.24722E+04	0.68901E+04	0.11660E+05
51	16	0.22980E+04	0.0	0.93624E+04	0.11660E+05
52	17	0.22980E+04	0.20012E+04	0.73612E+04	0.11660E+05

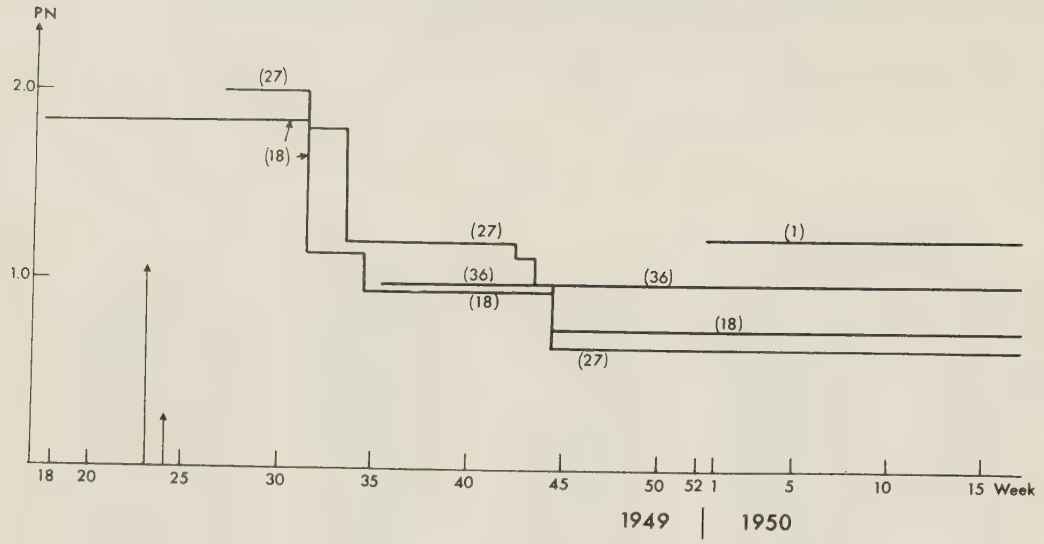

Fig 6a. Values of PN (normal electricity production) obtained from the optimization model and increases (illustrated by vertical arrows) made in the simulations. Time period starting at week 18 year 1949.
Unit: MWh.

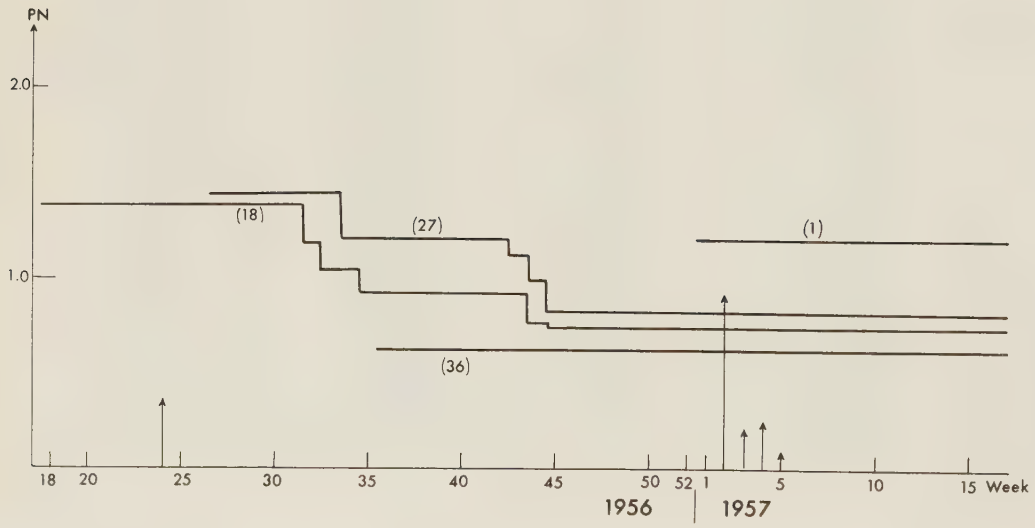

Fig 6b. Values of PN (normal electricity production) obtained from the optimization model and increases (illustrated by vertical arrows) made in the simulations. Time period starting at week 18 year 1956.
Unit: MWh.

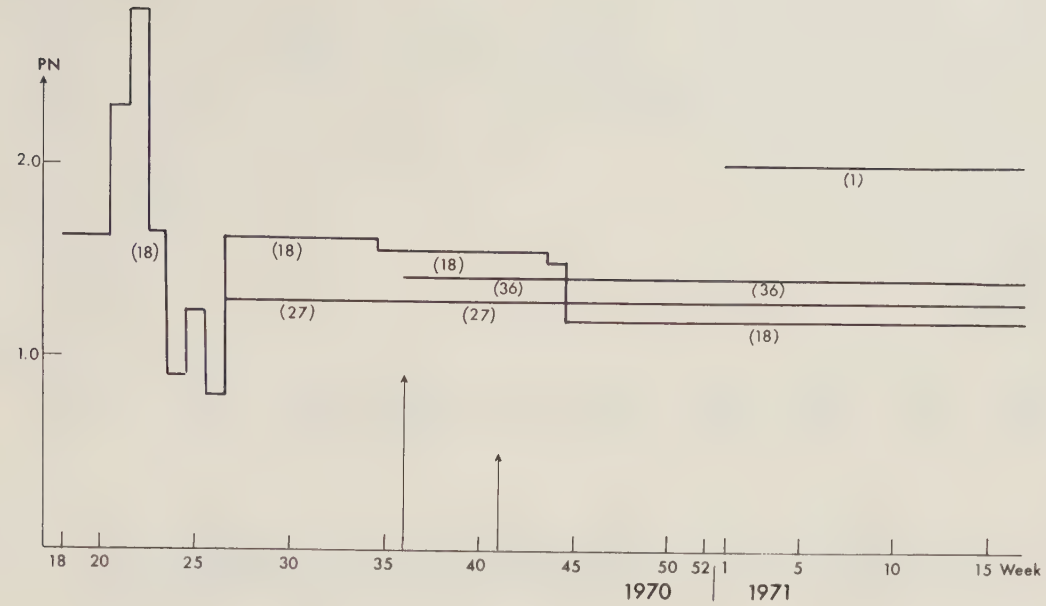

Fig 6c. Values of PN (normal electricity production) obtained from the optimization model and increases (illustrated by vertical arrows) made in the simulations. Time period starting at week 18 year 1970.
Unit: MWh.

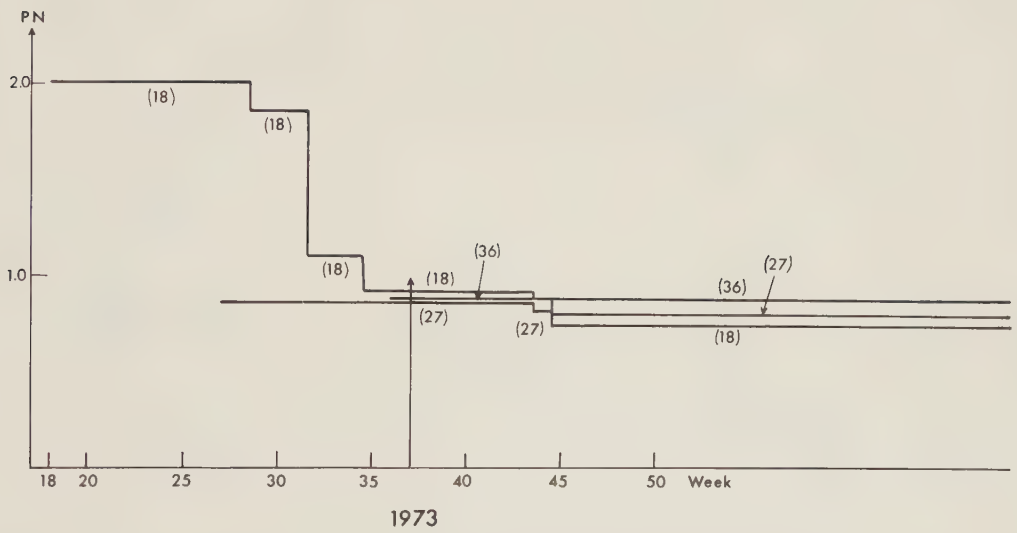

Fig 6d. Values of PN (normal electricity production) obtained from the optimization model and increases (illustrated by vertical arrows) made in the simulations. Time period starting at week 18 year 1973.
Unit: MWh.

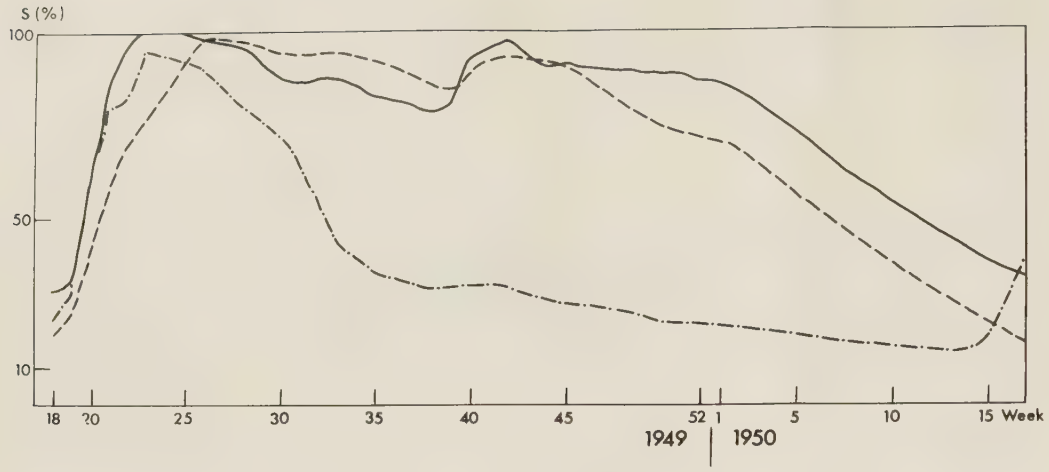

Fig 7a. Storages in percent of Smax that would result from our approach. Time period starting at week 18 year 1949.

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Fig 7b. Storages in percent of Smax that would result from our approach. Time period starting at week 18 year 1949.

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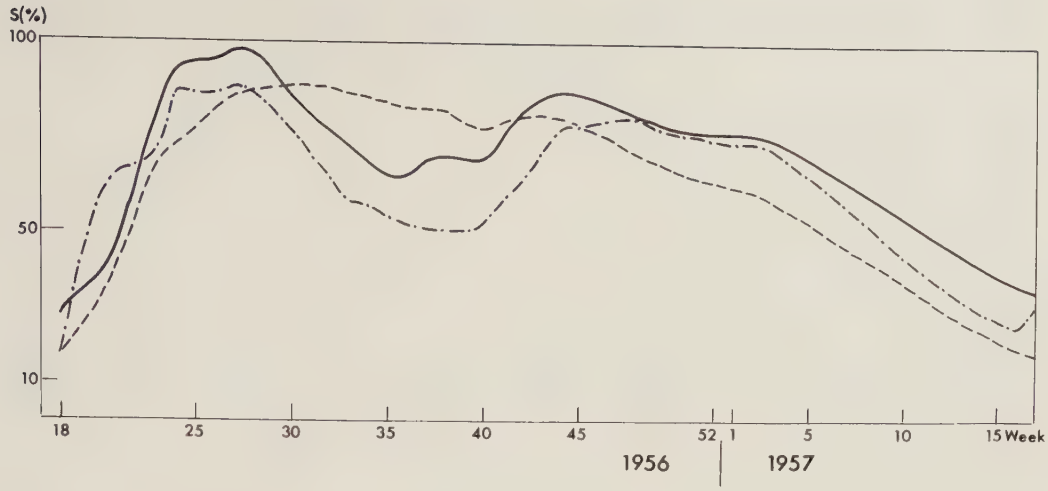

Fig 7c. Storages in percent of Smax that would result from our approach. Time period starting at week 18 year 1956.

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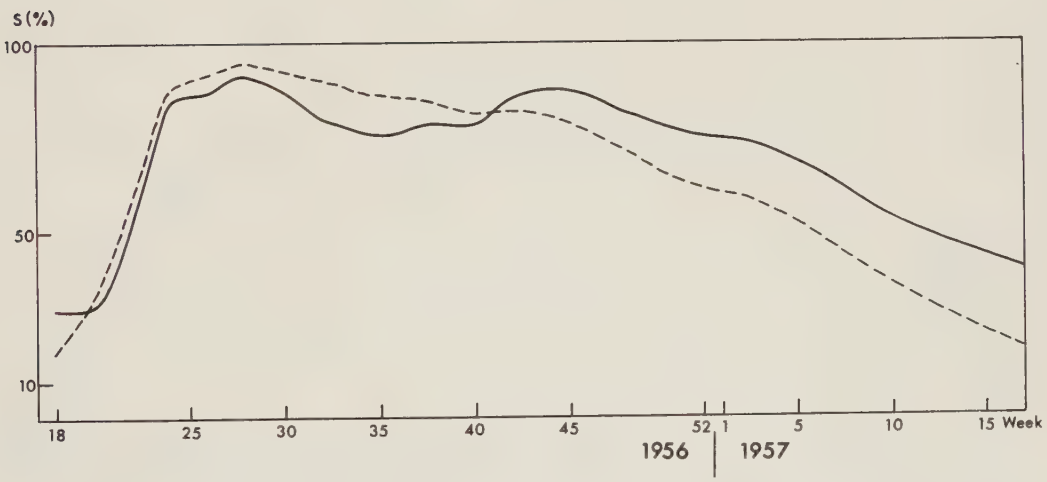

Fig 7d. Storages in percent of Smax that would result from our approach. Time period starting at week 18 year 1956.

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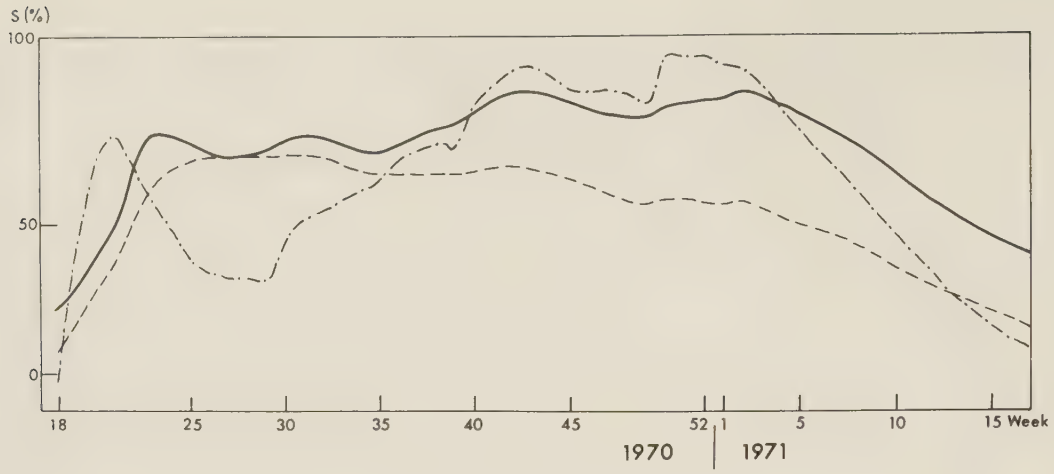

Fig 7e. Storages in percent of Smax that would result from our approach. Time period starting at week 18 year 1970.

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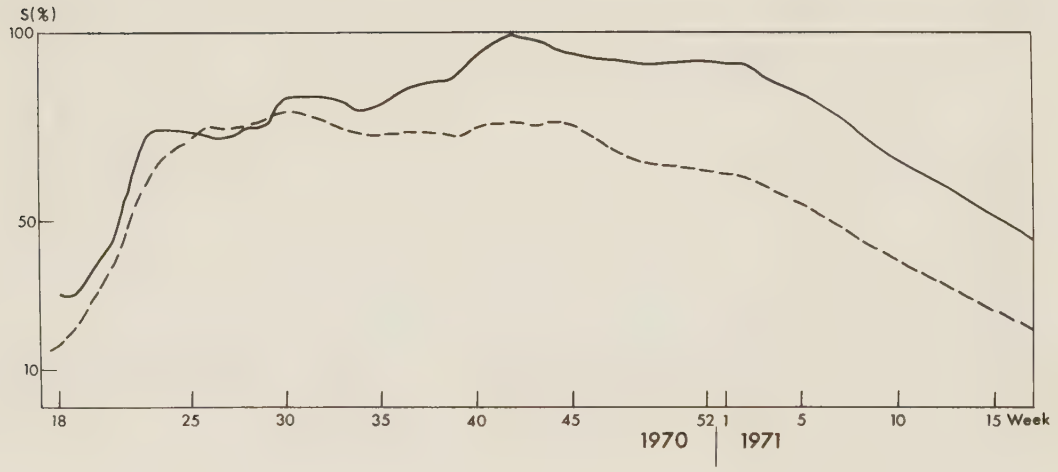

Fig 7f. Storages in percent of Smax that would result from our approach. Time period starting at week 18 year 1970.

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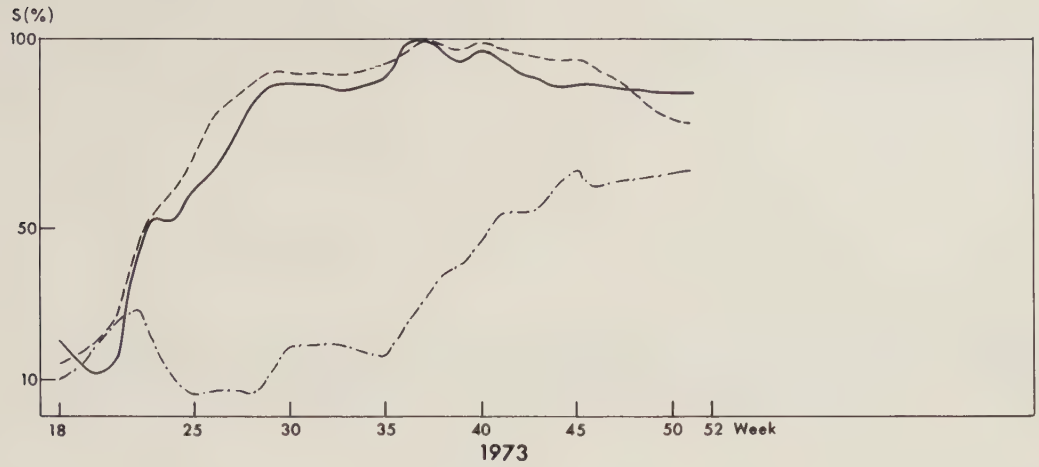

Fig 7g. Storages in percent of Smax that would result from our approach. Time period starting at week 18 year 1973.

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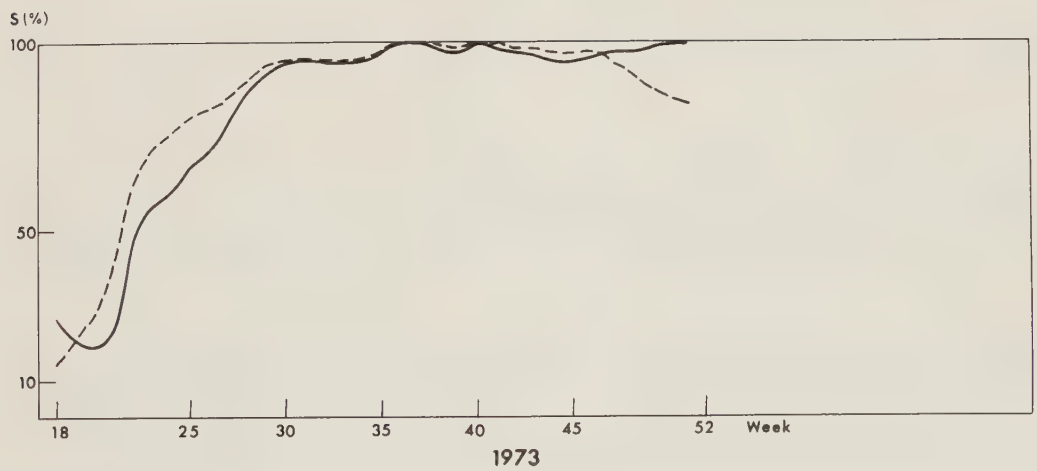

Fig 7h. Storages in percent of Smax that would result from our approach. Time period starting at week 18 year 1973.

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6 FINAL DISCUSSION

An implied assumption in our re-runnings of the optimization model is, that new energy contracts may be signed several times a year, where the new PN obtained is thought of as the new contract.

Another approach in case of plenty of water would be to apply an optimization model of the same type as eqs (15) - (18) extended to our 5-reservoir system. (In the discussion below this new model is called OPT2, while the former optimization model is called OPT1.) Then the elements of the matrix X obtained from OPT1 should be used as lower bounds on the releases, and the market price on non-firm energy should be the prices. I.e. in this approach we try to maximize the benefit of selling surplus power.

As stated in conjunction with eqs (15) - (18) this model has the serious drawback, that the releases may be very erratic, with maximum release one time step and minimum (minimum=lower bound) the next time step. Moreover, it is questionable if the policy derived with OPT2 is a proper use of our hydro-electricity system because, if the other power systems also try to follow the same optimization approach, the model would try to produce very much power for the time steps with the highest prices and no power (in excess of the contracts) in the other time steps.

OPT2 may also be applied (after appropriate modifications) to the case with shortage of water, but also in this case it has the same serious drawback.

A third alternative is to simply increase (decrease) releases (and power production in case of a plant below the dam) in times of storage above (below) pre-determined limits, for example in the way applied in our simulation model.

How often re-optimization with OPT1 should be applied in our combined optimization-simulation approach, is a matter of discussion. Maybe it should not only be applied at certain times of the year, but also in case of certain states of our system (reservoir storages, inflows, etc).

In the computations we have used one single table of B_k (Table 2) describing the relative demand, although in reality also the vector B varies. In time of the re-computations we probably have new and updated values of B_k, which can be used. Running our model OPT1 with different values of B presents no problems however.

A difficult problem is the choices of α and β. By increasing β or α we may switch from the condition

$$PN2 > PN1$$

to

$$PN2 < PN1$$

which means that "full regulation" (see section 5.4) may be possible. Every increase in α or β increases the number of corrections we probably have to apply in our simulations. The idea of using a very high β, is to rely heavily on the probabilities of high inflow to be followed by high inflow and on the possibilities to make corrections in our release scheme. Because, if high inflow were followed by normal or low flows, and we in the weeks with high inflows had increased the predetermined power production by selling surplus power, the risk of not being able to produce the contracted energy would increase.

By increasing α we obtain higher PN, but also the risk of not being able to produce energy according to the contract increases.

One approach to the choice of α would be an economic analysis of the benefits of higher PN and the expected losses due to inability (or our system) to generate energy according to the contract.

Finally it is again noticed, that this is just a study of a simplified system in a simplified environment (power market, electrical engineering, natural environment etc). But it is the opinion of the author, that the science of stochastic hydrology has something to contribute to the management of hydro-electricity systems.

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SOME DEFINITIONS AND ABBREVIATIONS

ESDP	Explicit Stochastic Dynamic Programming, see sec. 2.2.
ISDP	Implicit Stochastic Dynamic Programming, see sec. 2.2.
LDR	Linear Decision Rule, see p 22
LP	Linear Programming.
NLP	Non-Linear Programming.
QP	Quadratic Programming.
SRP	Static Release Policy, see p 26
VE	The amount of water corresponding to 1 m^3/s during one week.

APPENDIX 1. COMPUTATION PROCEDURES FOR TWO-RESERVOIR PROBLEMS

The optimization model described in sections 5.2 and 5.4 has straightforward solutions for two-reservoir serial systems if feasible solutions exist for PN = PN1. Here we only discuss feasibility conditioned on PN=PN1. Two cases are described here:

i) One power-plant which is situated below the lower reservoir (Problem A1).
ii) Two power-plants, one below each of the two reservoirs (Problem A2).

Let us start with the first case, see fig A1!

Problem A1

$$\text{Max } PN \tag{A1}$$

subject to

$$P_k = B_k \cdot PN \tag{A2}$$

$$P_k = H(2) \cdot X(2,k) \tag{A3}$$

$$(k = 1,2,\ldots,NT)$$

$$\sum_{k=1}^{T} X(1,k) \leq S(1,0)+QS^{*\alpha}(1,T) \tag{A4}$$

$$\sum_{k=1}^{T} (-X(1,k)+X(2,k)) \leq S(2,0)+QS^{*\alpha}(2,T) \tag{A5}$$

$$\sum_{k=1}^{T} X(1,k) \geq S(1-0)-Smax(1,T)+QS^{*(1-\beta)}(1,T) \tag{A6}$$

$$\sum_{k=1}^{T} (-X(1,k)+X(2,k)) \geq S(2,0)-Smax(2,T)+QS^{*(1-\beta)}(2,T) \tag{A7}$$

$$(T=1,2,3,\ldots,NT)$$

$$Xmin(1,k) \leq X(1,k) \leq Xmax(1,k) \quad \text{(A8)}$$

$$Xmin(2,k) \leq X(2,k) \leq Xmax(2,k) \quad \text{(A9)}$$

$(k=1,2,3,.....,NT)$

NT Number of time steps in the planning period

H(I) head of plant below reservoir I.

Otherwise the symbols is the same as in section 5.2.

For simplicity we have assumed that Smin=0 for both reservoirs, otherwise Smin would have appeared in the equations as in section 5.2. This simplification does not change the procedure however.

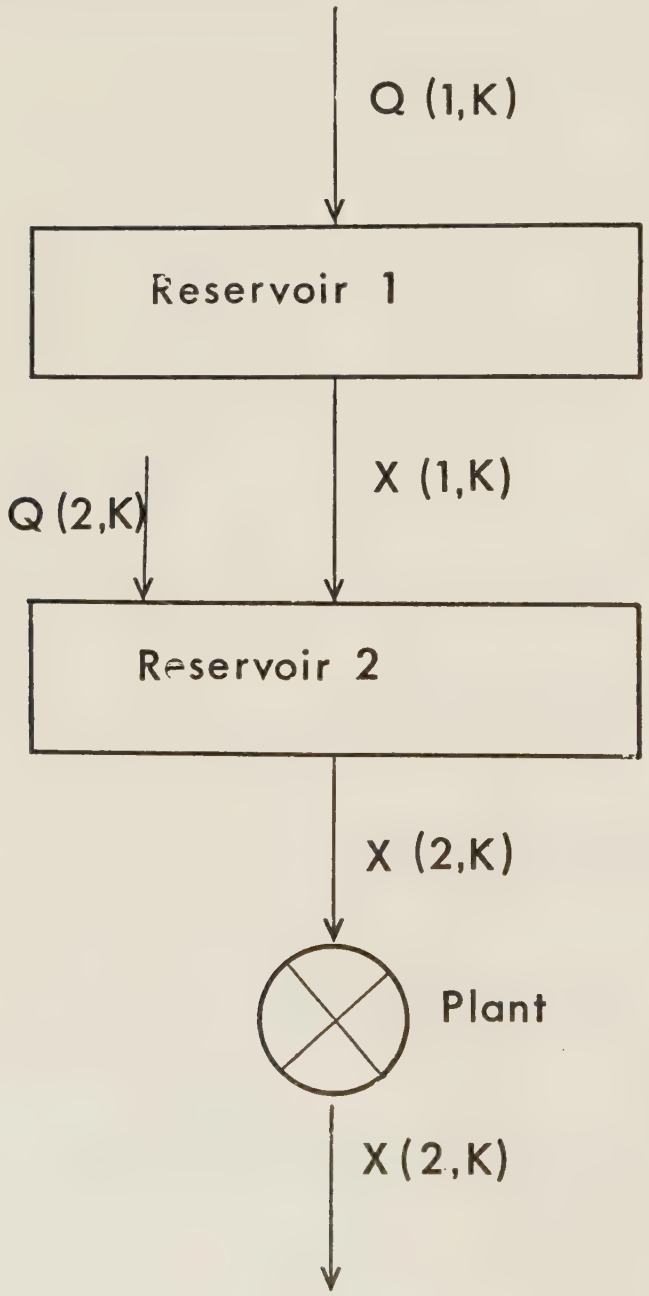

Fig A1. Reservoir system for problem A1.

Inserting eq (A3) in eq (A2) and summing over the time steps we obtain

$$PN \sum_{k=1}^{T} B_k = H(2) \cdot \sum_{k=1}^{T} X(2,k) \tag{A10}$$

Eqs (A4) + (A5) $\Longrightarrow$

$$\sum_{k=1}^{T} X(2,k) \leq \sum_{I=1}^{2} S(I,0) + QS^{*\alpha}(I,T)) \tag{A11}$$

Substituting eq (A11) into eq (A10) we obtain

$$PN \sum_{k=1}^{T} B_k \leq H(2) \cdot \left[\sum_{I=1}^{2} (S(I,0) + QS^{*\alpha}(I,T)) \right] \tag{A12}$$

And in the same way as in section 5.4 we compute

$$PN1 = \underset{1 \leq T \leq NT}{\text{Min}} \left[H(2) \cdot \left(\sum_{I=1}^{2} (S(I,0) + QS^{*\alpha}(I,T)) \right) / \left(\sum_{k=1}^{T} B_k \right) \right] \tag{A13}$$

Choosing PN=PN1 and using eqs (A3) and (A2), X(2,k) is determined by

$$H(2) \cdot X(2,k) = B_k \cdot H(2) \cdot \left(\sum_{I=1}^{2} (S(I,0) + QS^{*\alpha}(I,TT)) \right) / \left(\sum_{k=1}^{TT} B_k \right)$$

or

$$X(2,k) = B_k \cdot \left(\sum_{I=1}^{2} (S(I,0) + QS^{*\alpha}(I,TT)) \right) / \left(\sum_{k=1}^{TT} B_k \right) \tag{A14}$$

where TT is the critical time step considering shortage, compare section 5.4. In the same way as in this section PN2 is computed, and a necessary but not sufficient condition for a feasible solution to exist is

$$PN2 \leq PN1$$

Another condition is that X(2,k) computed by eq (A14) satisfies eq (A9). If this condition is satisfied, we try to find a feasible sequence X(1,k); if we succeed, we have found an optimal and feasible solution. In the search for this sequence we consider all X(2,k) as constants according to eq (A14). Starting with the restrictions on storage, we combine eqs (A5) and (A7), (A4) and (A6) respectively and rearrange.

$$\sum_{k=1}^{T} X(2,k) - S(2,0) - QS^{*\alpha}(2,T) \leq \sum_{k=1}^{T} X(1,k) \leq$$

$$\sum_{k=1}^{T} X(2,k) - S(2,0) - QS^{*(1-\beta)}(2,T) + Smax(2,T) \tag{A15}$$

$$S(1,0)-Smax(1,T)+QS^{*\,(1-\beta)}(1,T) \leq \sum_{k=1}^{T} X(1,k) \leq S(1,0)+QS^{*\,\alpha}(1,T) \qquad (A16)$$

Eq (A15) expresses the restrictions on storage for reservoir 2 and eq (A16) for reservoir 1.

These two equations are of the same kind, and are combined to

$$MVL(T) \leq \sum_{k=1}^{T} X(1,k) \leq MHL(T) \qquad (A17)$$

where MVL(T) is the maximum of the two right hand sides (eqs A15 and A16) and MHL(T) is the minimum of the two left hand sides. A necessary condition for feasibility is

$$MVL(T) \leq MHL(T) \qquad (A18)$$

To summarize, we must find a sequence X(1,k) which satisfies

$$Xmin(1,k) \leq X(1,k) \leq Xmax(1,k) \qquad (A8)$$

$$MVL(T) \leq \sum_{k=1}^{T} X(1,k) \leq MHL(T) \qquad (A17)$$

A subroutine in Fortran IV to compute the sequence X(1,k) is listed. Here the subscript denoting reservoir number is dropped, thus X(1,k) ⟶ X(k) etc. The procedure starts by putting X equal to Xmin and then increases X by the minimum amounts required to satisfy the left of the inequalities in eq (A17). When increasing X we check that we never obtain X greater than Xmax. If we cannot satisfy the left inequality in eq (A17) without violating the right one or violating X ≤ Xmax, our problem is infeasible.

```
      SUBROUTINE A1(X,XMIN,XMAX,NT,MVL,MHL)
      REAL MVL,MHL
      DIMENSION X(NT),XMIN(NT),XMAX(NT),SX(NT),MVL(NT),MHL(NT)
C     SX(K) IS THE SUM OF X FROM TIME STEP ONE TO (INCLUDING)
C     TIME STEP K.
      DO 1 K=1,NT
      X(K)=XMIN(K)
      SX(K)=X(K)
      IF(K.EQ.1)GO TO 1
      SX(K)=SX(K)+SX(K-1)
    1 CONTINUE
      DO 2 K=1,NT
      IF(SX(K).GE.MVL(K)) GO TO 2
      KK=K
    6 D=MVL(K)-SX(K)
      D1=XMAX(KK)-X(KK)
      D=AMIN1(D,D1)
      DO 4 L=KK,NT
      D2=MHL(L)-SX(L)
```

```
      D=AMIN1(D,D2)
    4 CONTINUE
      IF(D.LE.0) GO TO 8
      X(K)=(K)+D
      DO 5 L=KK,NT
    5 SX(L)=SX(L)+D
      IF(SX(K).GE.MVL(K)) GO TO 2
    8 IF(KK.EQ.1) GO TO 7
      KK=KK-1
      GO TO 6
    2 CONTINUE
      DO 3 K=1,NT
      IF(SX(K).GT.MHL(K)) GO TO 7
    3 CONTINUE
      RETURN
    7 PRINT 100
  100 FORMAT (10X,'PROBLEM INFEASIBLE')
      STOP
      END
```

The problem solved by the subroutine may be considered a network problem, see fig A2! Here Xmin and Xmax, MVL and MHL are the minimum and maximum "flows" on the arcs corresponding to X and SX respectively. In this subroutine KK is the time step where X is increased. D2 is the maximum amount that X can be increased without violating $SX \leq MHL$.

If $MVL(K) > SX(K)$, we have to increase X for time step K or one of the earlier time steps, see fig A2. We start by putting KK=K and try to obtain MVL(K)=SX(K). If necessary we decrease KK step by step until this condition is obtained. If this is impossible the problem is infeasible. The choice of the time steps KK ensures that a minimum number of SX is changed.

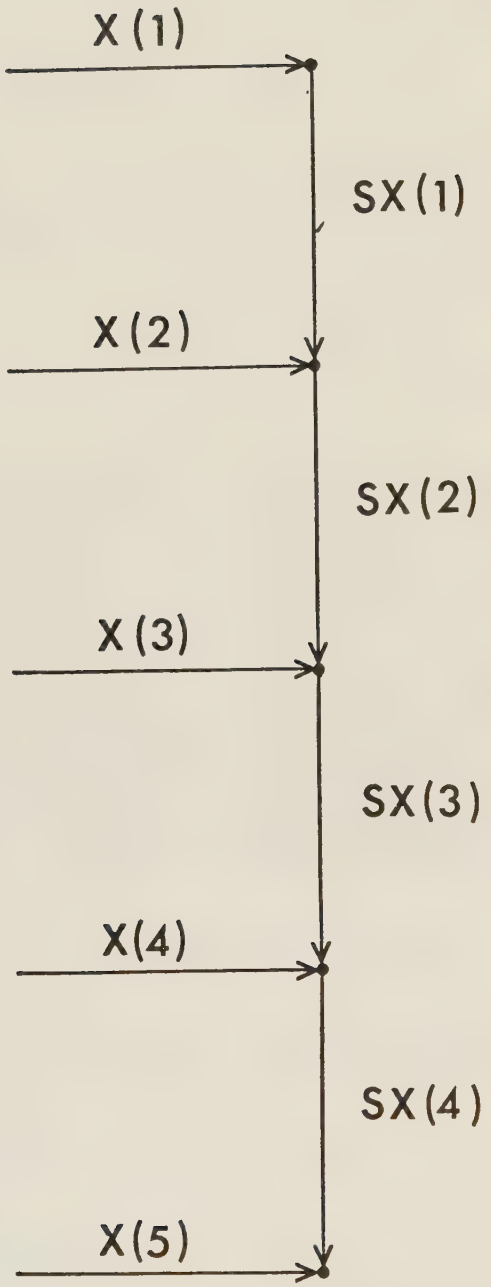

Fig A2. Part of network describing the problem solved in subroutine A1

Turning to our two-plant system, see figure A3 below!

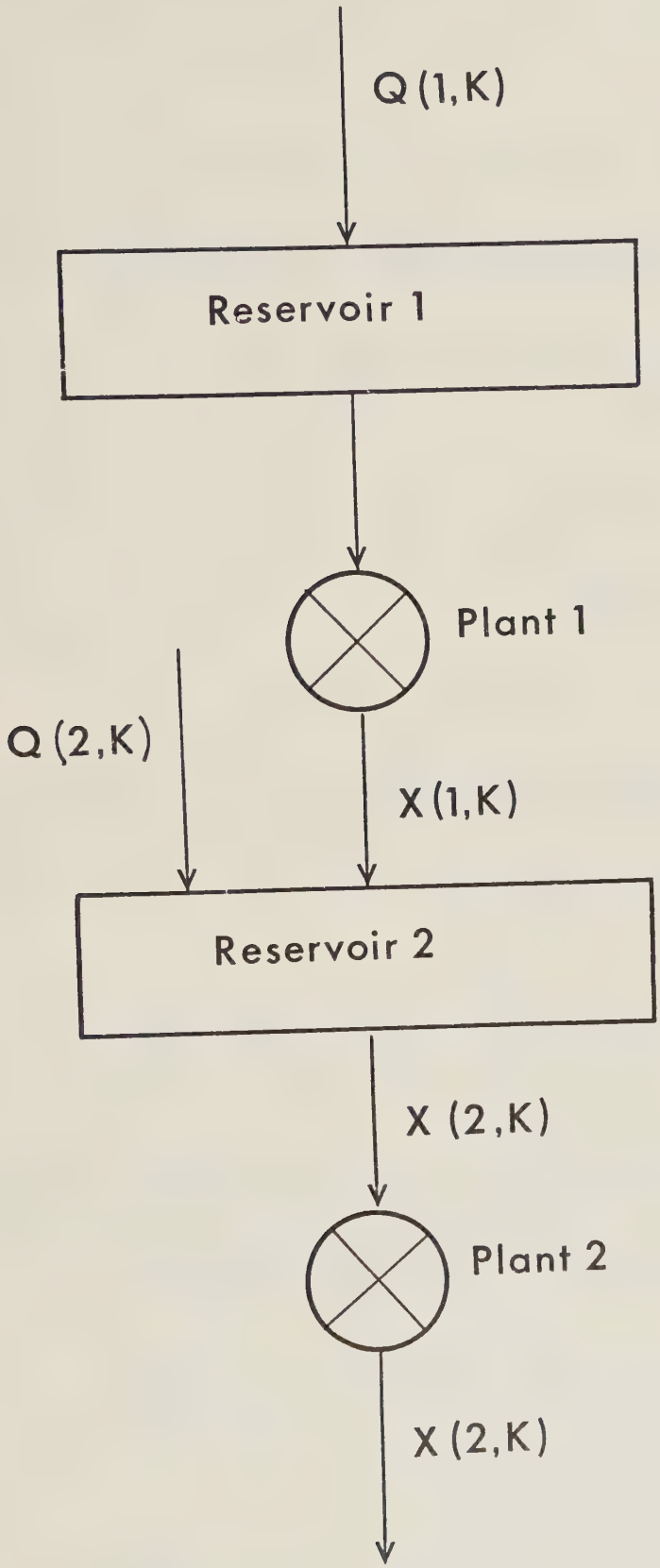

Fig A3. System for problem A2.

Problem A2

$$\text{Max PN} \tag{A19}$$

Subject to

$$P_k = B_k \cdot PN \tag{A20}$$

$$P_k = H(1) \cdot X(1,k) + H(2) \cdot X(2,k) \tag{A21}$$

and equations (A4)-(A9) which are unchanged for this problem. Inserting eq (A20) in eq (A21) and summing over the time steps we have

$$PN \sum_{k=1}^{T} B_k = H(1) \cdot \sum_{k=1}^{T} X(1,k) + H(2) \cdot \sum_{k=1}^{T} X(2,k) \tag{A22}$$

We also have (as in problem A1)

$$\sum_{k=1}^{T} X(2,k) \leq \sum_{I=1}^{2} (S(I,0)+QS^{*\alpha}(I,T)) \tag{A11}$$

Inserting eqs (A11) and (A4) into eq (A22) we obtain

$$PN \sum_{k=1}^{T} B_k \leq H(1) \cdot (S(1,0)+QS^{*\alpha}(1,T)) + H(2) \cdot \left(\sum_{I=1}^{2} (S(I,0)+QS^{*\alpha}(I,T)) \right)$$

And also for this system we compute the "PN1"

$$PN1 = \underset{1 \leq T \leq NT}{\text{Min}} \left[\left((H(1) \cdot (S(1,0)+QS^{*\alpha}(1,T)) + H(2) \cdot \left(\sum_{I=1}^{2} (S(I,0)+QS^{*\alpha}(I,T)) \right) \right) / \left(\sum_{k=1}^{T} B_k \right) \right] \tag{A23}$$

As in the other cases, we do not know if a feasible solution with PN=PN1 exists. But, as done in the previous system, we only discuss if a feasible solution with this choice of PN exists. Also here we define TT as the time which gives PN1 from eq (A23). PN2 is computed by

$$PN2 = \underset{1 \leq T \leq NT}{\text{Max}} \left[(H(1) \cdot (S(1,0)-Smax(1,T)+QS^{*(1-\beta)}(1,T)) + H(2) \left(\sum_{I=1}^{2} (S(I,0)-Smax(I,T)+QS^{*(1-\beta)}(I,T)) \right) / \left(\sum_{k=1}^{T} B_k \right) \right]$$

One condition for feasibility is

$$PN2 \leq PN1$$

Eqs (A20) and (A21) with PN=PN1 gives

$$H(1)\cdot X(1,k)+H(2)\cdot X(2,k)=B_k\cdot PN1$$

or

$$X(1,k)=(PN1\cdot B_k-H(2)\cdot X(2,k))/H(1) \tag{A24}$$

This inserted into eq (A8), the constraint on releases for reservoir 1 gives

$$Xmin(1,k) \leq (PN1\cdot B_k-H(2)\cdot X(2,k))/H(1) \leq Xmax(1,k) \tag{A25}$$

Rearranging we obtain

$$(PN1\cdot B_k-H(1)\cdot Xmax(1,k))/H(2) \leq X(2,k) \leq$$

$$(PN1\cdot B_k-H(1)\cdot Xmin(1,k))/H(2) \tag{A26}$$

This constraint is of the same form as eq (A9) and therefore they are combined into

$$X'min(2,k) \leq X(2,k) \leq X'max(2,k) \tag{A27}$$

where we of course in all cases use the strongest restrictions. This assures that eqs (A8) and (A9) will both be satisfied. The fact that X(1,k) is completely determined once X(2,k) is chosen, makes it possible to express also the restrictions on storage in X(2,k). From eq (A24) we may write

$$\sum_{k=1}^{T} X(1,k)=(PN1\cdot \sum_{k=1}^{T} B_k-H(2)\cdot \sum_{k=1}^{T} X(2,k))/H(1) \tag{A28}$$

Now equations (A4) and (A6) are combined and eq (A28) inserted which gives the restrictions on storage for reservoir 1 expressed in X(2,k).

$$S(1,0)-Smax(1,T)+QS^{*(1-\beta)}(1,T)$$

$$\leq (PN1\cdot \sum_{k=1}^{T} B_k-H(2)\cdot \sum_{k=1}^{T} X(2,k))/H(1) \leq$$

$$S(1,0)+QS^{*\alpha}(1,T)$$

Or

$$(PN1 \sum_{k=1}^{T} B_k-H(1)\cdot (S(1,0)+QS^{*\alpha}(1,T)))/H(2) \leq \sum_{k=1}^{T} X(2,k) \leq \tag{A29}$$

$$(PN1 \sum_{k=1}^{T} B_k - H(1) \cdot (S(1,0) - Smax(1,T) + QS^{*(1-\beta)}(1,T)))/H(2) \qquad \text{(A29 cont)}$$

In the same way the storage restrictions for reservoir 2 may be expressed in X(2,k). Combining eqs (A5) and (A7) gives

$$S(2,0) - Smax(2,T) + QS^{*(1-\beta)}(2,T) \leq \sum_{k=1}^{T} (-X(1,k) + X(2,k))$$

$$\leq S(2,0) + QS^{*\alpha}(2,T)$$

Inserting eq (A28) we obtain

$$S(2,0) - Smax(2,T) + QS^{*(1-\beta)}(2,T)$$

$$\leq \left((H(1)+H(2)) \cdot \sum_{k=1}^{T} X(2,k) - PN1 \cdot \sum_{k=1}^{T} B_k \right) / H(1) \leq$$

$$S(2,0) + QS^{*\alpha}(2,T)$$

Or

$$\left[H(1) \cdot (S(2,0) - Smax(2,T) + QS^{*(1-\beta)}(2,T)) + PN1 \cdot \sum_{k=1}^{T} B_k \right] / (H(2)+H(1))$$

$$\leq \sum_{k=1}^{T} X(2,k) \leq \qquad \text{(A30)}$$

$$\left[H(1) \cdot (S(2,0) + QS^{*\alpha}(2,T)) + PN1 \cdot \sum_{k=1}^{T} B_k \right] / (H(2)+H(1))$$

Equations (A29) and (A30) are of the same kind and thus they are combined (where we of course choose the strongest restrictions).

$$MIS(T) \leq \sum_{k=1}^{T} X(2,k) \leq MAS(T) \qquad \text{(A31)}$$

Summarizing, our problem is to find a sequence

$$X(2,k)$$

satisfying

$$X'min(2,k) \leq X(2,k) \leq X'max(2,k) \qquad \text{(A27)}$$

and

$$MIS(T) \leq \sum_{k=1}^{T} X(2,k) \leq MAS(T) \qquad \text{(A31)}$$

But this is a problem of quite the same kind as that arising from problem A1! Thus the same subroutine may be used. After X(2,k) is computed, X(1,k) is determined by eq (A24).

APPENDIX 2. RESULTS OF PROBABALISTIC FORECASTING

In this appendix some results of probabalistic forecasting with our stochastic models of inflow are displayed in some figures. The quantity which is forecasted is

$$QS = \sum Q$$

for Stora Mjölkvattnet and Korsvattnet. The summation starts at a certain week, which is noticed in the figures. In most figures two forecasts are shown, of which one is with assumed known conditions at the end of the last week preceeding the first week of the summation. Displayed in the figures are the QS that really occurred and $QS^{*0.10}$, $QS^{*0.50}$ and $QS^{*0.90}$ for the different forecasts. The unit is VE.

Fig A4 a. Probabalistic forecasting of sums of inflow to Stora Mjölkvattnet. Summation starts at week 18 year 1949.
Forecast - - - - - - - - - -
Record ——————

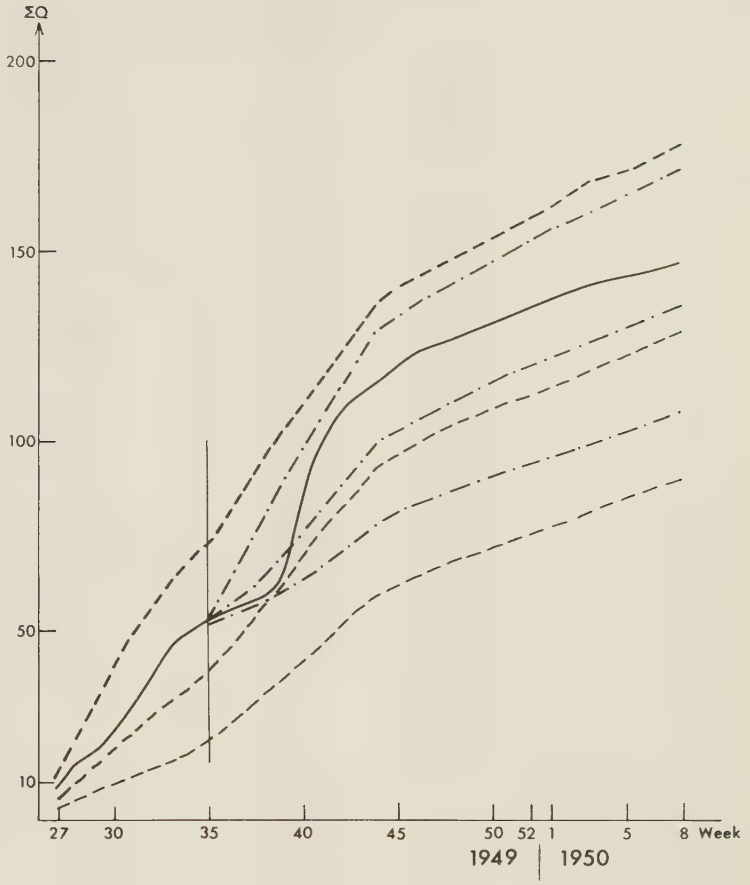

Fig A4 b. Probabalistic forecasting of sums of inflow to Stora Mjölkvattnet. Summation starts at week 27 year 1949.
Forecast issued at end of week 26 - - - - - - -
Forecast issued at end of week 35 -.-.-.-.-.-.-
Record ——————

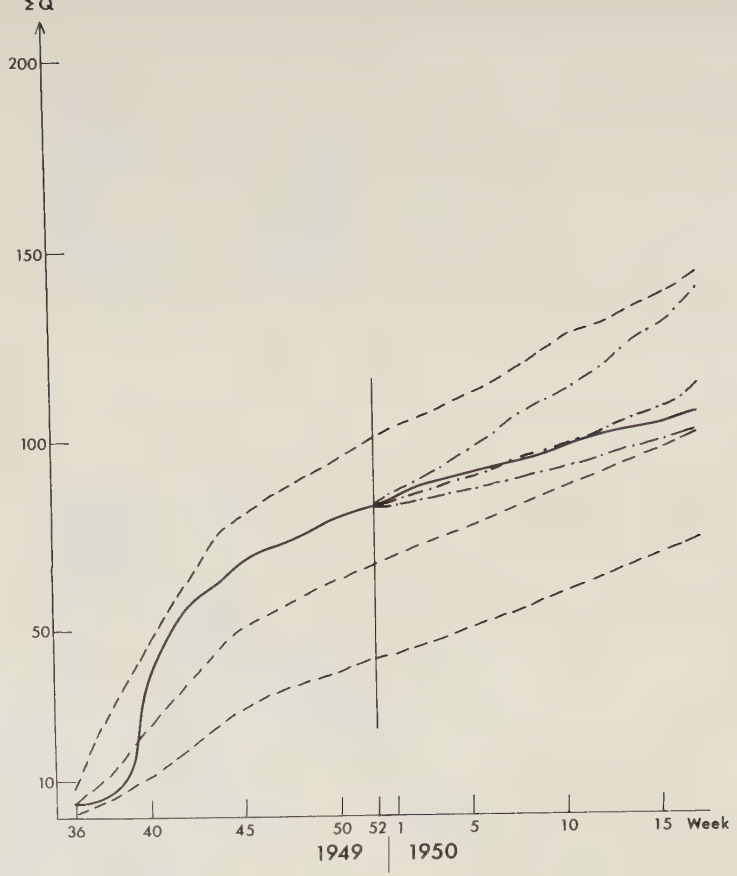

Fig A4 c. Probabalistic forecasting of sums of inflow to Stora Mjölkvattnet. Summation starts at week 36 year 1949.
Forecast issued at end of week 35 - - - - - - - - - - -
Rorecast issued at end of week 52 -.-.-.-.-.-.-.-.-.-.-
Record ————————————————————

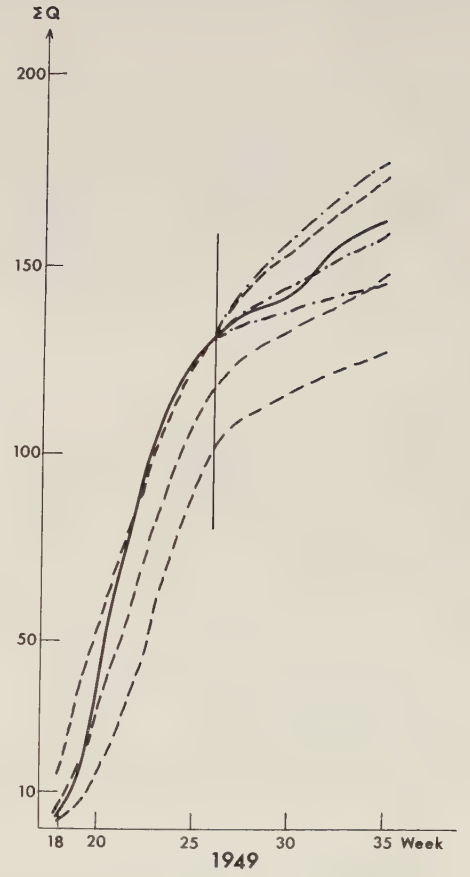

Fig A4 d. Probabalistic forecasting of sums of inflow to Korsvattnet. Summation starts at week 18 year 1949.
Forecast issued at end of week 17 - - - - - - -
Forecast issued at end of week 26 -.-.-.-.-.-.-.-
Record————————————————————

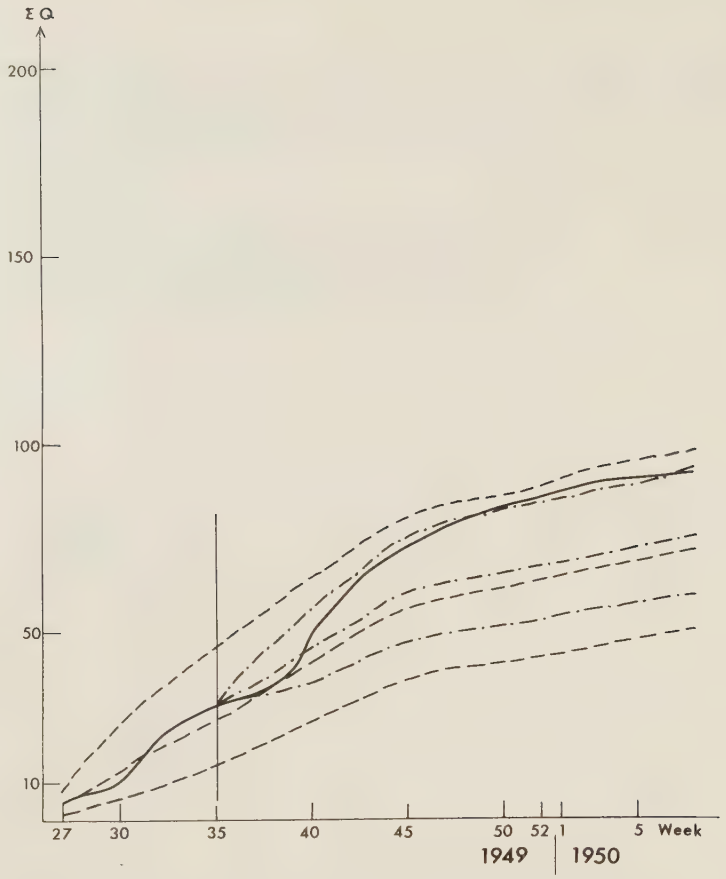

Fig A4 e. Probabalistic forecasting of sums of inflow to Korsvattnet. Summation starts at week 27 year 1949.
Forecast issued at end of week 26 - - - - - - - - - -
Forecast issued at end of week 35 -.-.-.-.-.-.-.-.-.-
Record

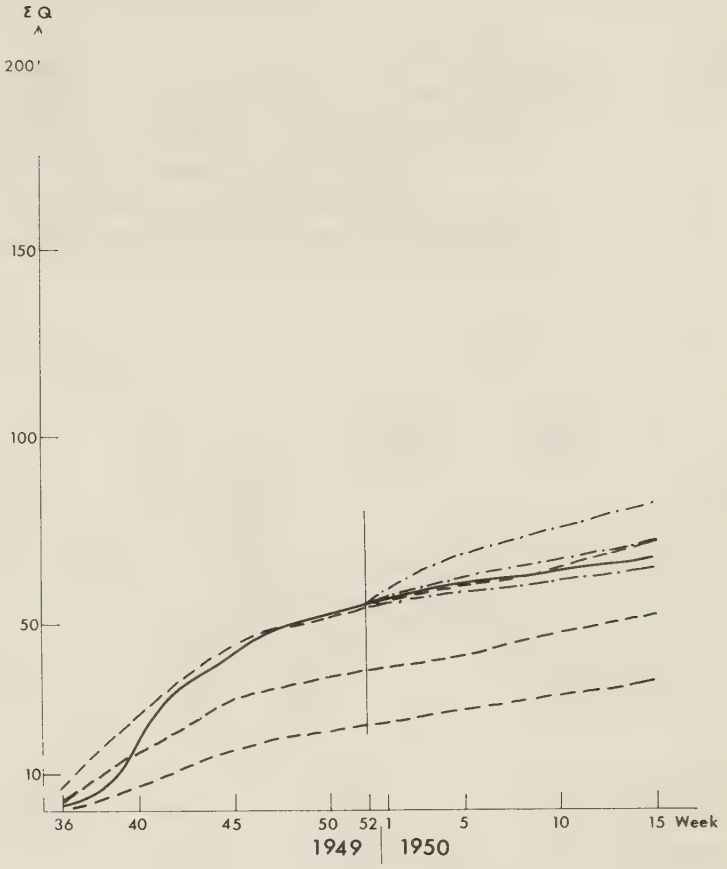

Fig A4 f. Probabalistic forecasting of sums of inflow to Korsvattnet.
Summation starts at week 36 year 1949.
Forecast issued at end of week 35 - - - - - - - - - -
Forecast issued at end of week 52 -.-.-.-.-.-.-.-.-.-
Record

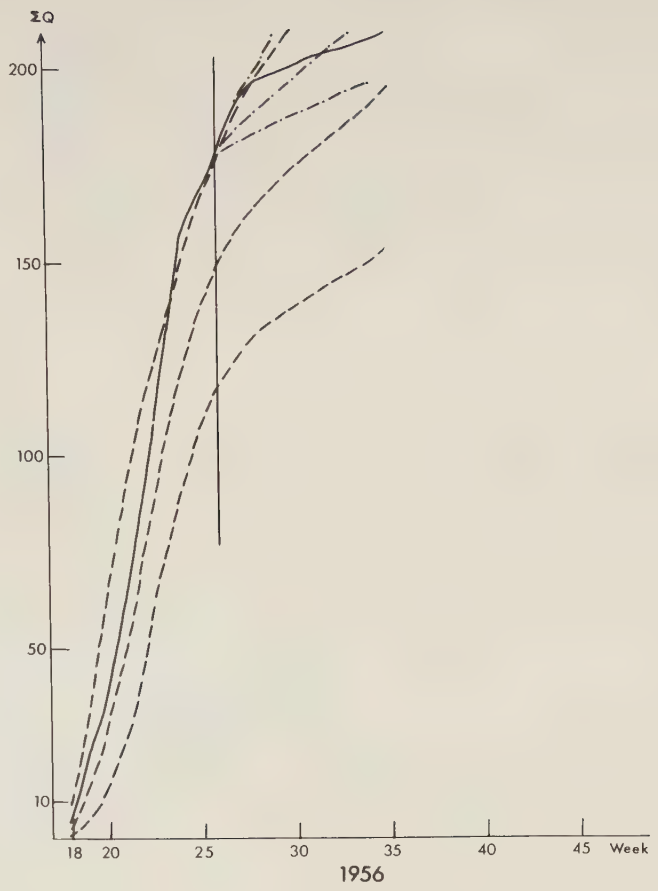

Fig A4 g. Probabalistic forecasting of sums
of inflow to Stora Mjölkvattnet.
Summation starts at week 18 year 1956.
Forecast issued at end of week 17 - - - - - - - - - - -
Forecast issued at end of week 26 -.-.-.-.-.-.-.-.-.-.-
Record

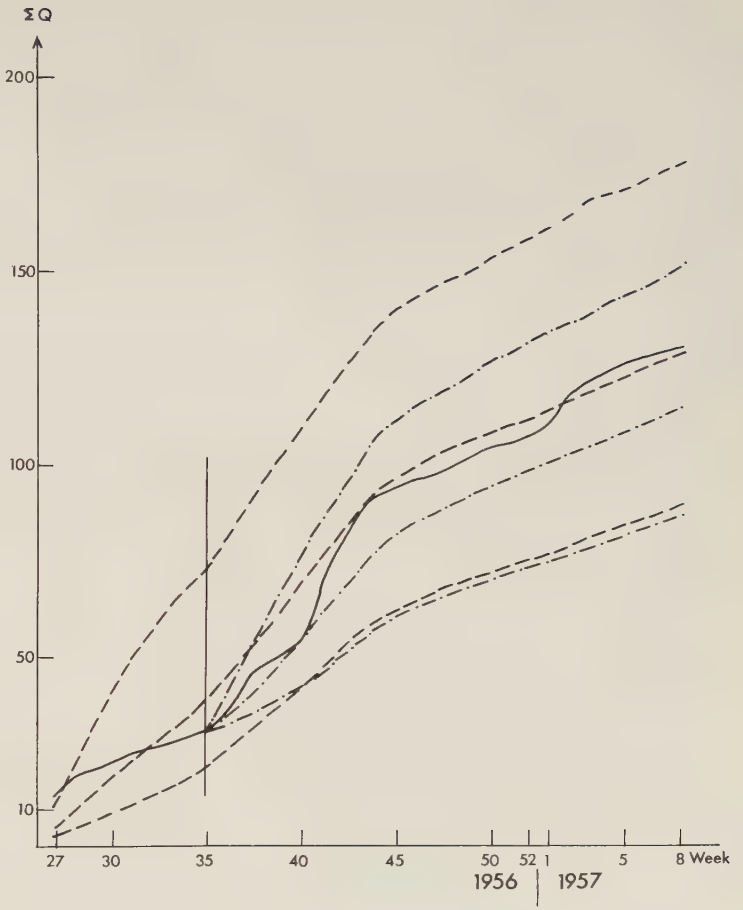

Fig A4 h. Probabalistic forecasting of sums
of inflow to Stora Mjölkvattnet.
Summation starts at week 27 year 1956.
Forecast issued at end of week 26 - - - - - - -
Forecast issued at end of week 35 -.-.-.-.-.-.-.-
Record

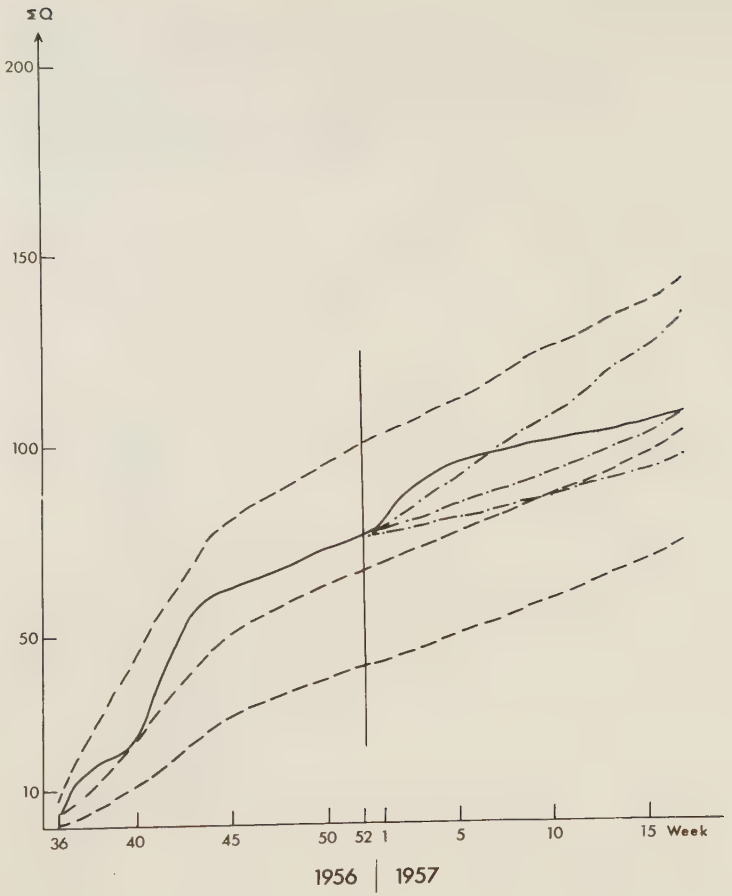

Fig A4 i. Probabalistic forecasting of sums
of inflows to Stora Mjölkvattnet.
Summation starts at week 36 year 1956.
Forecast issued at end of week 35 - - - - - - - - - - -
Forecast issued at end of week 52 -.-.-.-.-.-.-.-.-.-.-
Record

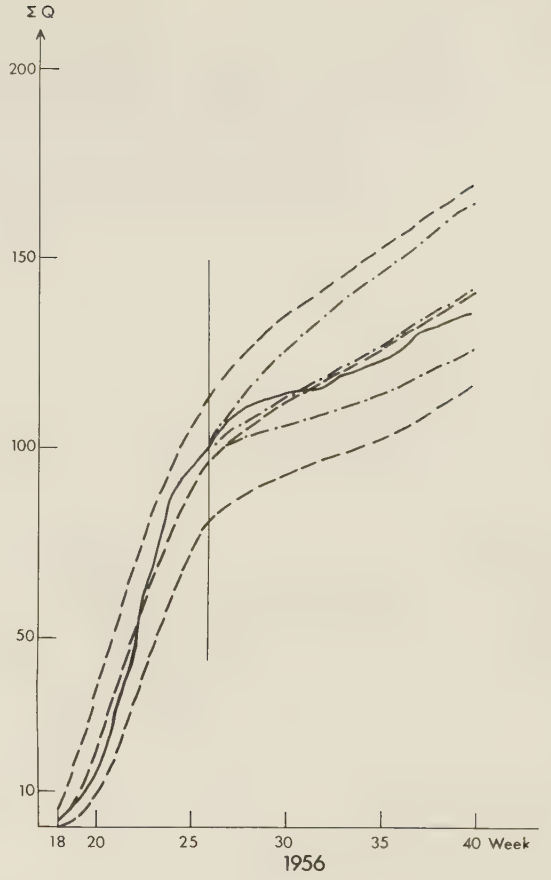

Fig A4 j. Probabalistic forecasting of sums
of inflow to Korsvattnet.
Summation starts at week 18 year 1956.
Forecast issued at end of week 17 - - - - - - -
Forecast issued at end of week 26 -.-.-.-.-.-.-
Record

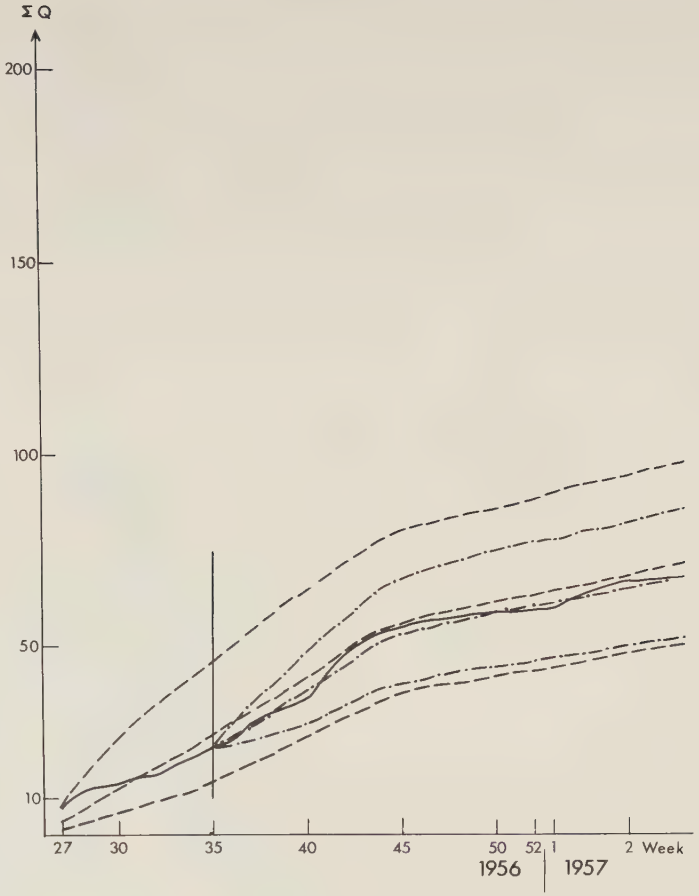

Fig A4 k. Probabalistic forecasting of sums
of inflows to Korsvattnet.
Summation starts at week 27 year 1956.
Forecast issued at end of week 26 - - - - - - - - - - -
Forecast issued at end of week 35 -.-.-.-.-.-.-.-.-.-.-
Record

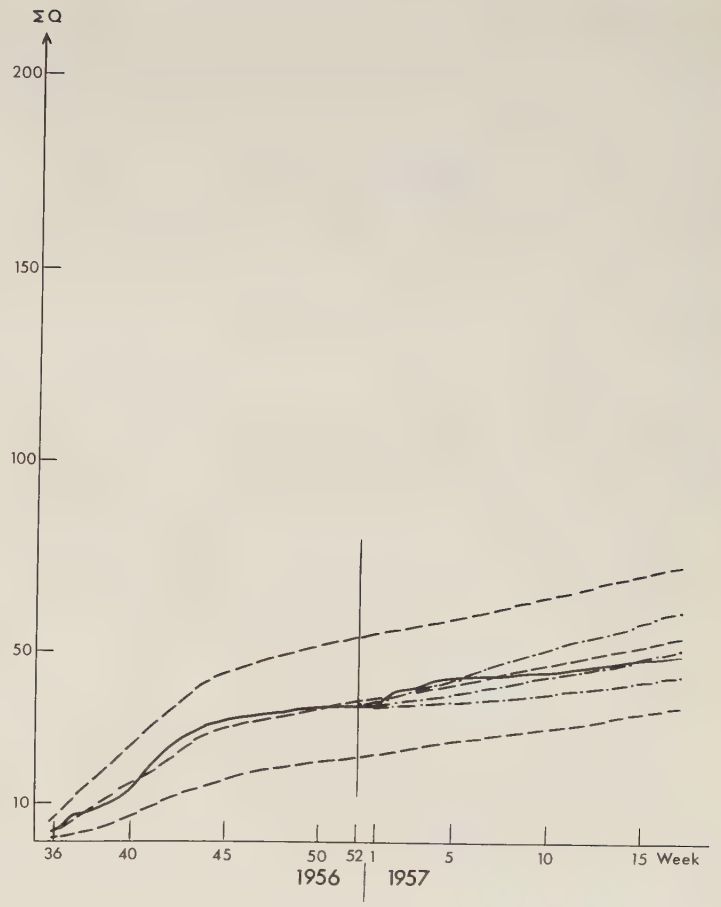

Fig A4 l. Probabalistic forecasting of sums
of inflows to Korsvattnet.
Summation starts at week 36 year 1956.
Forecast issued at end of week 35 - - - - - - -
Forecast issued at end of week 52 -.-.-.-.-.-.-.-
Record

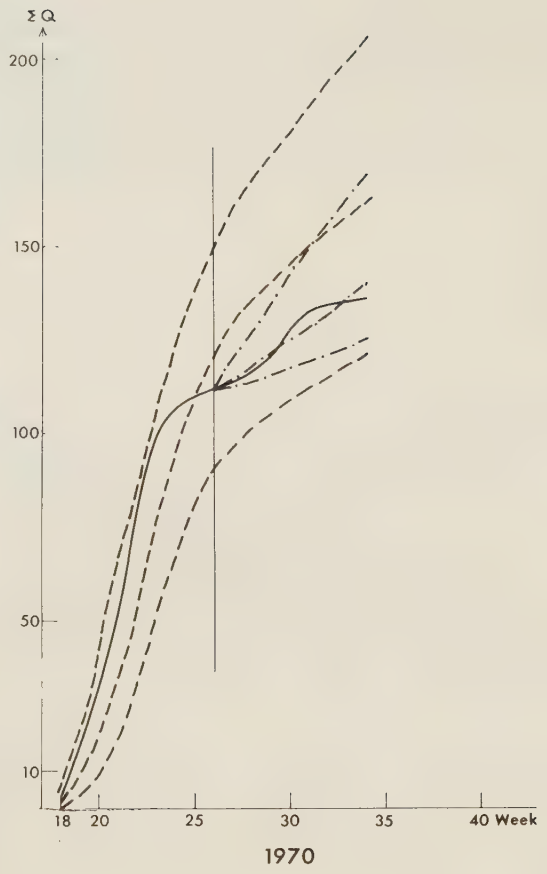

Fig A4 m. Probabalistic forecasting of sums
of inflows to Stora Mjölkvattnet.
Summation starts at week 18 year 1970.
Forecast issued at end of week 17 - - - - - - - - - -
Forecast issued at end of week 26 -.-.-.-.-.-.-.-.-.-
Record ———————

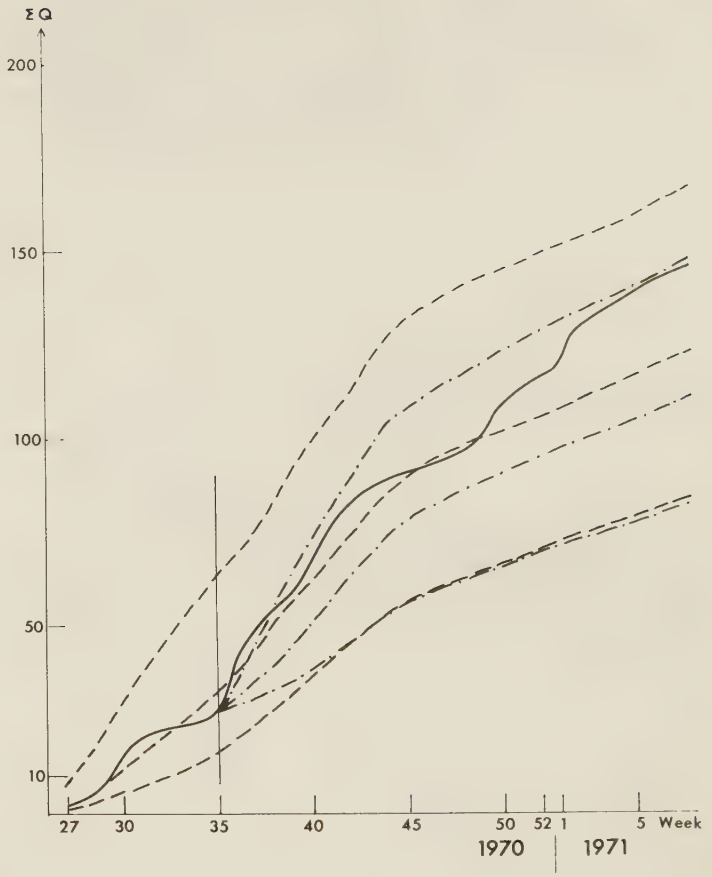

Fig A4 n. Probabalistic forecasting of sums
of inflow to Stora Mjölkvattnet.
Summation starts at week 27 year 1970.
Forecast issued at end of week 26 - - - - - - -
Forecast issued at end of week 35 -.-.-.-.-.-.-
Record ———————

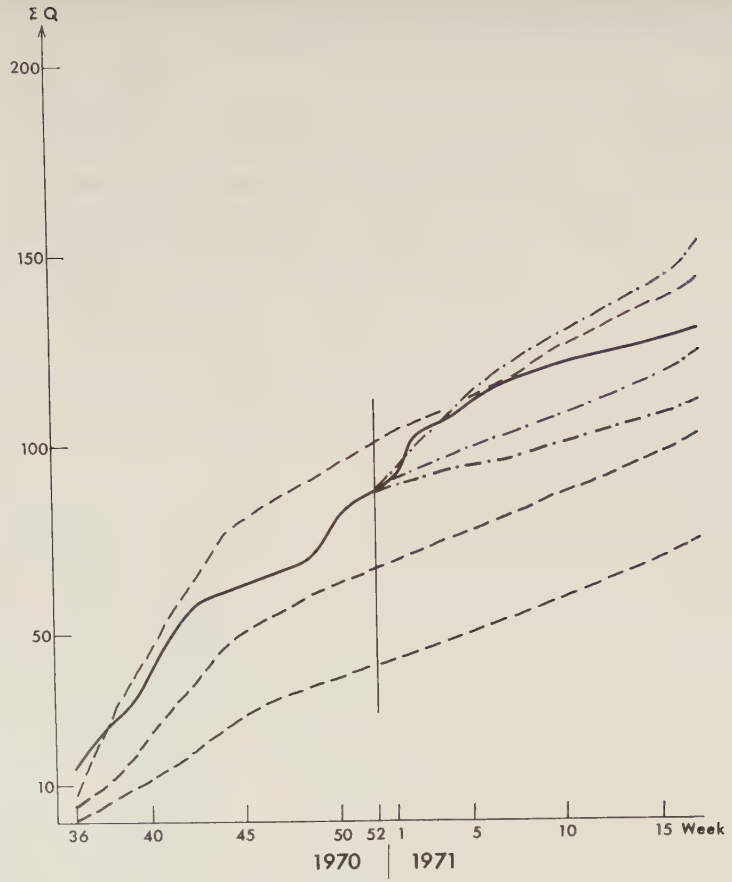

Fig A4 o. Probabalistic forecasting of sums
of inflow to Stora Mjölkvattnet.
Summation starts at week 36 year 1970.
Forecast issued at end of week 35 - - - - - - - - - -
Forecast issued at end of week 52 -.-.-.-.-.-.-.-.-.-.-
Record ———————

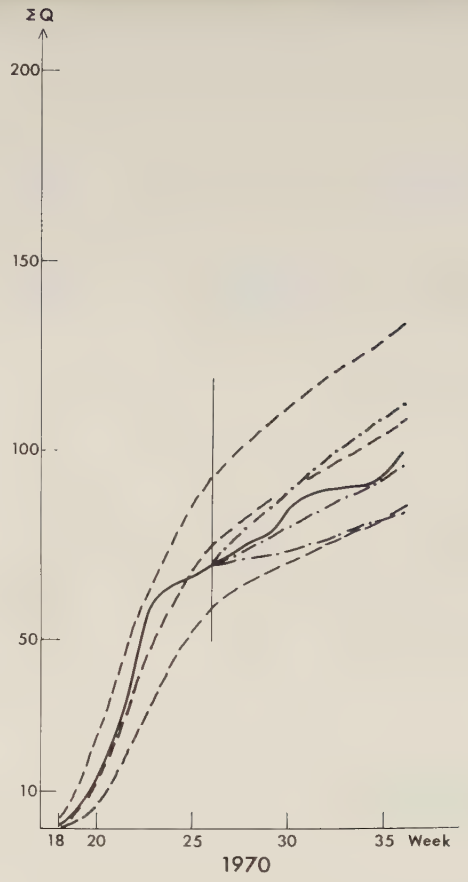

Fig A4 p. Probabalistic forecasting of sums
of inflow to Korsvattnet.
Summation starts at week 18 year 1970.
Forecast issued at end of week 17 - - - - - - -
Forecast issued at end of week 26 -.-.-.-.-.-.-
Record ———————

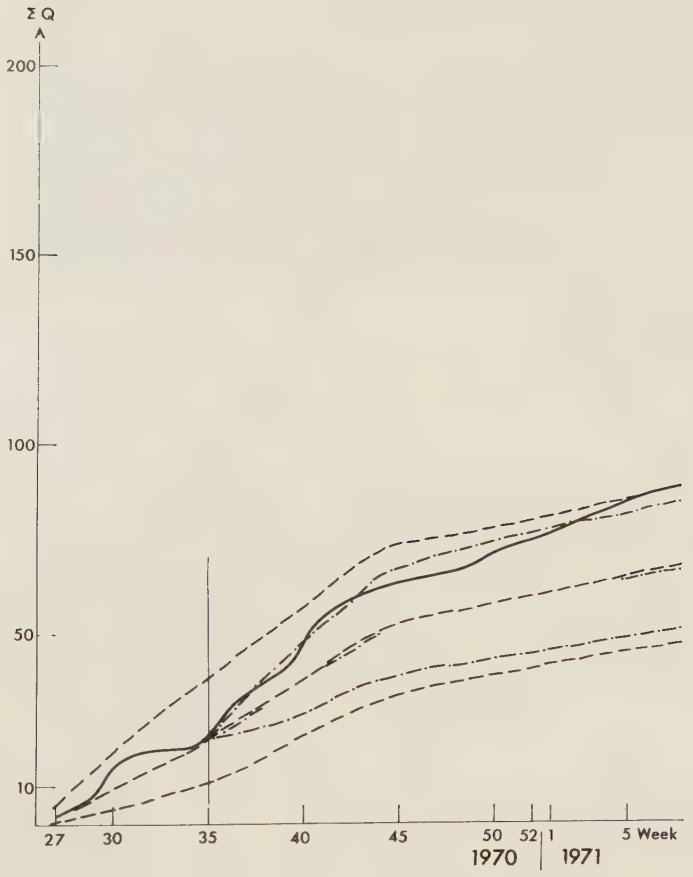

Fig A4 q. Probabalistic forecasting of sums
of inflow to Korsvattnet.
Summation starts at week 27 year 1970.
Forecast issued at end of week 26 - - - - - - - - - - -
Forecast issued at end of week 35 -.-.-.-.-.-.-.-.-.-.-
Record ———————

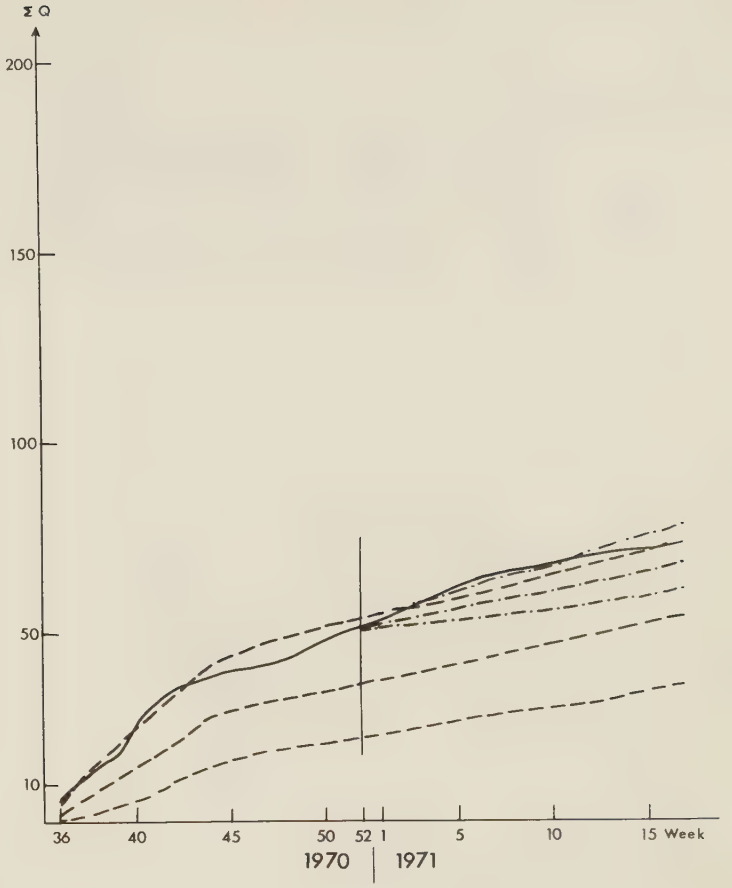

Fig A4 r. Probabalistic forecasting of sums
of inflow to Korsvattnet.
Summation starts at week 36 year 1970.
Forecast issued at end of week 35 - - - - - - -
Forecast issued at end of week 52 -.-.-.-.-.-.-
Record ———————

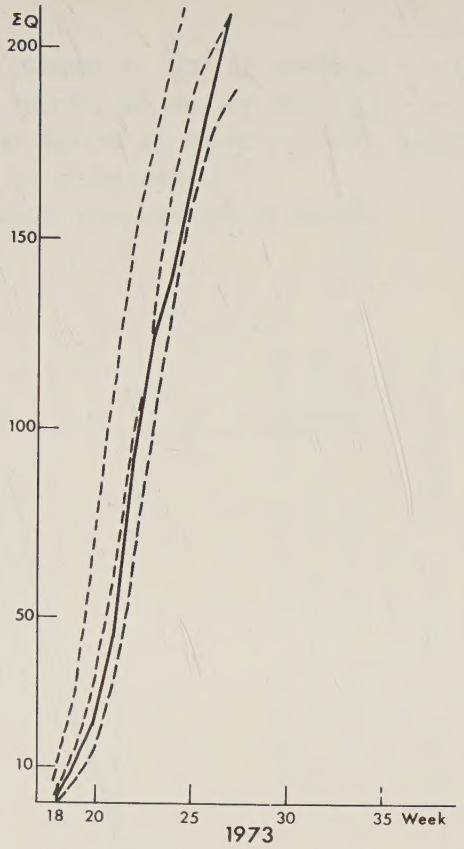

Fig A4 s. Probabalistic forecasting of sums of inflow to Stora Mjölkvattnet.
Summation starts at week 18 year 1973
Forecast - - - - - - - - - - -
Record ——————

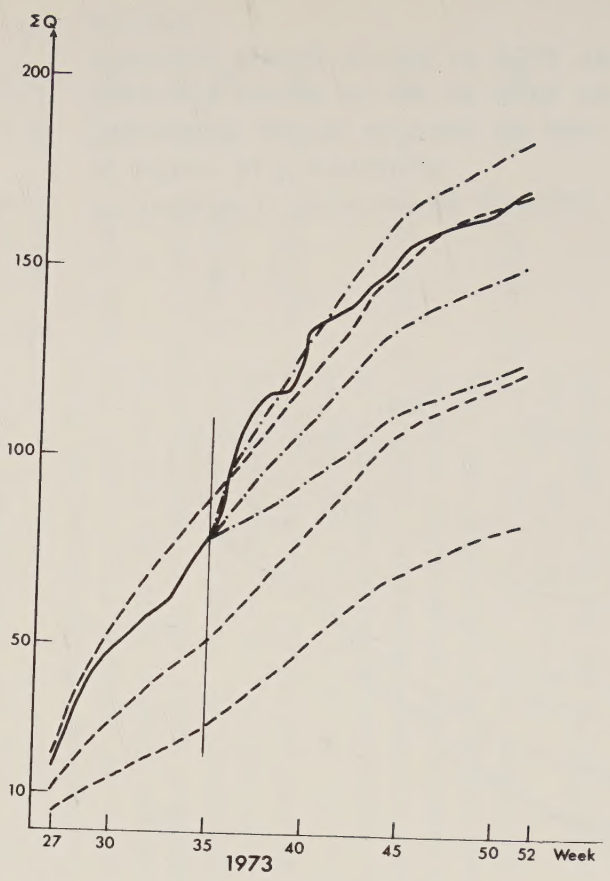

Fig A4 t. Probabalistic forecasting of sums of inflow to Stora Mjölkvattnet.
Summation starts at week 27 year 1973.
Forecast issued at end of week 26 - - - - - - - - - - -
Forecast issued at end of week 35 -.-.-.-.-.-.-.-.-.-.-
Record ——————

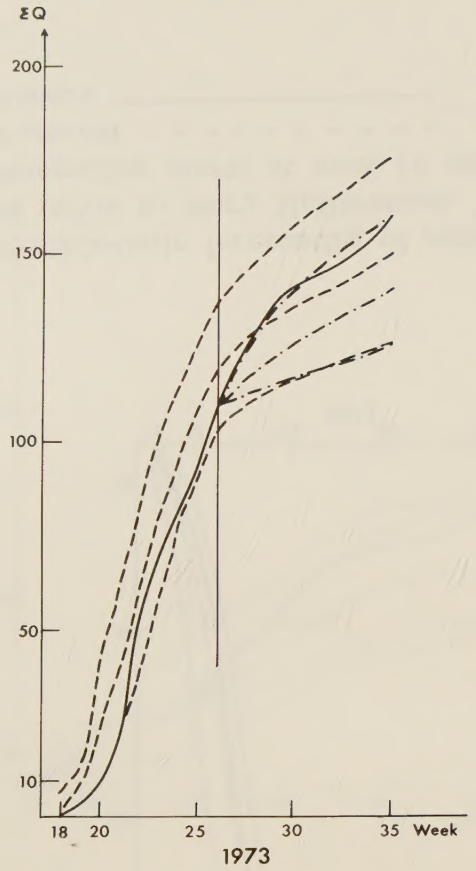

Fig A4 u. Probabalistic forecasting of sums
of inflow to Korsvattnet.
Summation starts at week 18 year 1973.
Forecast issued at end of week 17 - - - - - - - - - -
Forecast issued at end of week 26 -.-.-.-.-.-.-.-.-.-.-
Record ———————

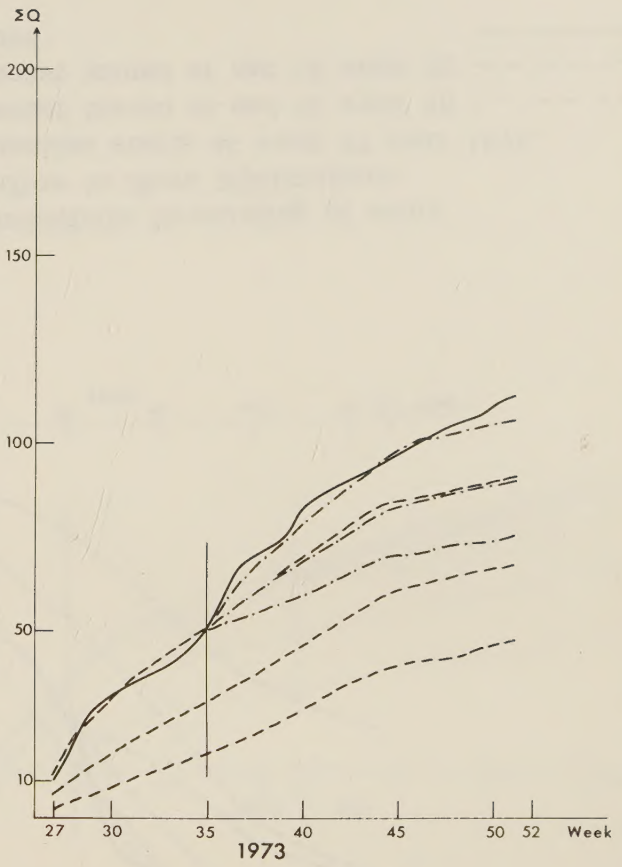

Fig A4 v. Probabalistic forecasting of sums
of inflow to Korsvattnet.
Summation starts at week 27 year 1973.
Forecast issued at end of week 26 - - - - - - - - -
Forecast issued at end of week 35 -.-.-.-.-.-.-.-.-
Record ———————

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